## Task 1 (PASS GRADE) – P3.1

**(A)** The power developed by Pelton Wheel water turbine is given by the following equation:

$$P = mu(v-u)(1-k\cos\theta)$$

Where

P is the power.

| | |
|---|---|
| m is the mass flow rate | m = 40 kg/s |
| u is the velocity of the buckets | |
| v is the velocity of the water jet | v = 20 m/s |
| k is the blade friction coefficient | k = 0.98 |
| $\theta$ is the angle of deflection | $\theta = 165^0$ |

Determine the velocity of the bucket that will make the power maximum and calculate the maximum power.
***(show that the value of the velocity obtained indeed maximises the power)***

**(B)** You are going to mount a light on the wall behind your desk. The light at a point S on the wall at height ***x*** foot illuminates a point P on the horizontal surface of the desk with intensity inversely proportional to the square of the distance from P to S and proportional to sine of the angle between the ray from P to S and the horizontal surface. Fix a point P on the desk 1 foot from the wall.
Show that the intensity is given by:

$$\frac{x}{(1+x^2)(\sqrt{1+x^2})}$$

Find the height of S above the desk for which the intensity at P is largest.

**(C)** An empty spherical vessel is filled with water. As the water level rises, the radius Of the water in the vessel and the volume of water will change. If the radius of water in the sphere increases at 0.5 m/s , find **using function of a function** differentiation rule ,the rate of change of volume when the radius is 5 cm

## Task 2 (PASS GRADE) – P3.2

**(A)** The velocity of a rocket is related to time by the following Laws:

1. $v = t^4 + \arcsin(t^2) + e^t \ln t + 7$
2. $v = \cos(\ln t) + \dfrac{t+1}{2t+3}$
3. $v = \arcsin(4t) + \arccos t$
4. $v = a\tanh(t^2)$
5. $v = t^2 - \ln(\sqrt{2}\ t) + \sinh(t^2 + 5)$

Determine $\dfrac{dv}{dt}$ and $\dfrac{d^2v}{dt^2}$ for the above five laws.

**(B)** The distance covered by a motorist as a function of time is given by :

$x = a\sinh(4t)$ Find $\dfrac{d^4x}{dt^4}$

## Task 3 (PASS GRADE) – P3.3

**(A)** A particle travels with a velocity of

v(t) = $3e^{-2t} \times t$ m/s.

What is the total distance , in meters, travelled by the particle during the time interval $0 \leq t \leq 2$ ?

**(B)** The expression of center of mass a homogenous plate of constant density $\rho$ in the shape of a semi-circle of radius r is given by:

$$\bar{x} = \frac{\int_0^r 2\rho\ x\sqrt{r^2 - x^2}\,dx}{\int_0^r 2\rho\sqrt{r^2 - x^2}\,dx}$$

Show that $\bar{x} = \frac{4}{3\pi}r$

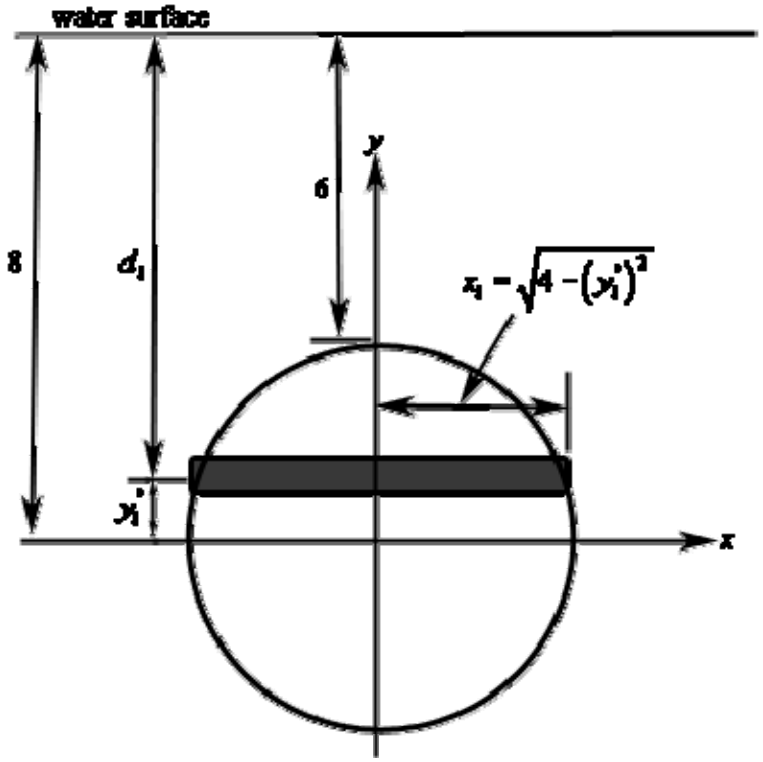
**(C)** The acceleration of a moving particle is related to time by the following Law:

$$a(t) = \frac{5t^2 - 2t + 13}{(t+3)(t-1)^2}$$

Determine an expression for the velocity $v(t)$ as a function of time.

## Task 4 (PASS GRADE) – P3.4

**(A)** Find the hydrostatic force on a circular plate of radius 2 that is submerged 6 meters in the water.




**(B)** A ball is thrown vertically upwards and after t seconds its height s(t) above the ground is given by

$$s(t) = 8 + 10t + 5t^2.$$

Find

(a) the height from which the ball is thrown

(b) the initial velocity of the ball

(c) the maximum height reached by the ball.