Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

ALGEBRA

Laws of Indices:

1. $a^m x\ a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{m.n}$
3. $a^{-n} = \dfrac{1}{a^n}$
4. $\dfrac{a^m}{a^n} = a^{m-n}$
5. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
6. $a^0 = 1$

Definition of a logarithm:

If $y = a^x$ then $x = log_a y$

Change of base:

$\text{Log}_a\, y = \dfrac{\log_b y}{\log_b a}$

Laws of logarithms:

1. $\log (A \text{ x } B) = \log A + \log B$
2. $\log\left(\dfrac{A}{B}\right) = \log A - \log B$
3. $\log A^n = n.\log A$

Quadratic Formula:

If $ax^2 + bx + c = 0$ Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Newton-Raphson Iterative Method:

If x_1 is the approximate value for the real root of the equation: f(x) = 0, then a closer approximation, x_2, can be obtained from: $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ Where $f'(x_1)$ is the gradient of the curve at x_1.

GRAPHS

Straight line graphs.

$$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$

Straight line law.

$y = mx + c$ m is the gradient.

c is where the line cuts the y axis

Least square method:

$$m\sum_{j=1}^{n} x_j^{\,2} + c\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} x_j y_j$$

$$m\sum_{j=1}^{n} x_j + nc = \sum_{j=1}^{n} y_j$$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

SERIES

Arithmetic series.

n th term $T_n = a + (n+1)d$ Sum of n terms $S_n = \frac{n}{2}(a+l) = \frac{n}{2}(2a + (n-1)d$

where a = first term, l = last term, d = difference between terms.

Geometric series.

n th term $T_n = ar^{n-1}$ Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$

Sum to infinity $S_\infty = \frac{a}{1-r}$

where a = first term, r = ratio between terms.

MacClaurin Series:

$$f(x) \approx f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots\ldots\ldots$$

Taylor Series:

$$f(x) \approx f(a) + \frac{(x-a)f'(a)}{1!} + \frac{(x-a)^2 f''(a)}{2!} + \frac{(x-a)^3 f'''(a)}{3!} + \frac{(x-a)^4 f^{iv}(a)}{4!} + \ldots\ldots\ldots$$

TRIGONOMETRY

Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\cos^2\theta + \sin^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $\cot^2\theta + 1 = \text{cosec}^2\theta$

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = +\cos\theta$ $\tan(-\theta) = -\tan\theta$

Compound angle addition and subtraction:

sin (A + B) = sin A cos B + cos A sin B sin (A - B) = sin A cos B - cos A sin B

cos (A + B) = cos A cos B - sin A sin B cos (A - B) = cos A cos B + sin A sin B

if R sin (kt + α) = a sin kt + b cos kt then: a = R cos α, b = R sin α,

$R^2 = (a^2 + b^2)$ and $\alpha = \tan^{-1}(b/a)$

Double angles:

Sin 2A = 2 sin A cos A

Cos 2A = $\cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Tan 2A = $\frac{2\tan A}{1-\tan^2 A}$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

MATRICES.

A matrix is an array of numbers: $\begin{pmatrix} 1 & -2 & 4 \\ 3 & 7 & 0 \end{pmatrix}$ is a 2 × 3 matrix

Adding $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$

Subtracting $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$

Multiplying $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$

Notice that when multiplying matrices, AB and BA are not necessarily equal.

Identity matrix I $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A \times A^{-1} = I$

Inverse of 2nd order matrix $A\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Determinant of 2nd order matrix $A\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $|A|$ or $\Delta_A = (ad) - (bc)$

Determinant of 3rd order matrix $B = \begin{pmatrix} P & Q & R \\ a & b & c \\ d & e & f \end{pmatrix}$ is

$$|B| \text{ or } \Delta_B = P\begin{vmatrix} b & c \\ e & f \end{vmatrix} - Q\begin{vmatrix} a & c \\ d & f \end{vmatrix} + R\begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

$$\therefore |B| \text{ or } \Delta_B = P\left[(bf)-(ec)\right] - Q\left[(af)-(dc)\right] + R\left[(ae)-(db)\right]$$

Cramer's rule

To solve simultaneous equations with 3 unknowns using determinants:

$$a_1x + b_1y + c\ z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0$$
$$a_3x + b_3y + c_3z + d_3 = 0$$

$$\frac{x}{\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}} = \frac{z}{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} = \frac{x}{\Delta_x} = \frac{-y}{\Delta_y} = \frac{x}{\Delta_z} = \frac{-1}{\Delta}$$

$$x = \frac{-\Delta x}{\Delta}$$
$$y = \frac{\Delta y}{\Delta}$$
$$z = \frac{-\Delta z}{\Delta}$$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

COMPLEX NUMBERS

Z = (a + jb) = r (cos θ + j sin θ) = r $\angle\,\theta$ = r $e^{j\theta}$ Where $j^2 = -1$

Modulus, r = | z | = $(a^2 + b^2)$ ***Argument***, θ = Tan^{-1} (b/a)

Addition: (a + jb) + (c + jd) = (a + c) + j(b + d)
Subtraction: (a + jb) - (c + jd) = (a - c) + j(b - d)

(Additions and subtractions cannot be performed in polar form)
Multiplication: $z_1z_2 = r_1r_2\angle(\theta_1 + \theta_2)$ ***Division***:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

Conversion

Polar $r\angle\theta$ to rectangular $a \pm jb$ $a = r\cos\theta,\ b = r\sin\theta$

Rectangular $a \pm jb$ to polar $r\angle\theta$

$r = \sqrt{a^2 + b^2}$ $\tan\theta = \left(\frac{b}{a}\right)$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

The complex conjugate of $(a + jb)$ ***is*** $(a - jb)$

Divide $\frac{a + jb}{c + jd}$ = $\frac{(a + jb)(c - jd)}{(c + jd)(c - jd)}$ (multiply top and bottom by complex conjugate of bottom)

$$= \frac{ac - jad + jbc - j^2bd}{c^2 - jcd + jcd - j^2d^2}$$

$$= \frac{(ac + bd) + j(-ad + bc)}{c^2 + d^2}$$

Powers $(r\angle\theta)^n = r^n\angle n\theta$ ***Roots*** $\sqrt[n]{r\angle\theta} = \sqrt[n]{r}\angle\frac{\theta}{n}$

De Moivre's Theorem: $(r\angle\,\theta)^n = r^n\angle n\,\theta = r^n(\cos n\,\theta + j\sin n\,\theta)$

GEOMETRY

Areas and Volumes.

Triangle Area = $\frac{1}{2}$ base x perpendicular height

$= \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

Parallelogram Area = base x perpendicular height

Trapezium Area = $\frac{1}{2}(a+b)h$

Circle Area = $\pi r^2 = \dfrac{\pi d^2}{4}$

Circumference = $2\pi r = \pi d$

Sphere Volume = $\dfrac{4}{3}\left(\pi r^3\right)$ Surface Area = $4\pi r^2$

Cylinder Volume = $\pi r^2 h$ Curved surface area = $2\pi rh$

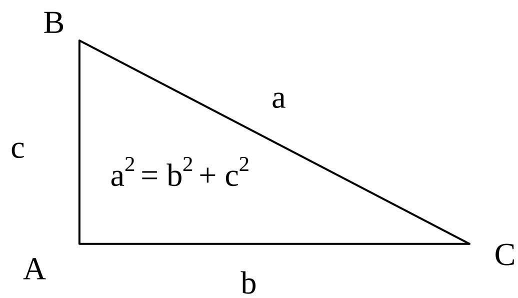
Cone Volume = $\dfrac{1}{3}\left(\pi r^2 h\right)$ where h = vertical height

Curved surface area = πrl where l = slant height

Prism Volume = base area x perpendicular height

Pyramid Volume = $\dfrac{1}{3}$(base area) x (perpendicular height)

Pythagoras's Theorem:



Circular Measure. 2π radians = 360^0

To convert radians to degrees multiply by $\dfrac{360}{2\pi}$, To convert degrees to radians multiply by $\dfrac{2\pi}{360}$

Length of arc = $r\theta$ where θ is in radians Area of sector = $\frac{1}{2}r^2\theta$

Non right-angled triangles

$$\frac{\mathbf{a}}{\mathbf{\sin A}} = \frac{\mathbf{b}}{\mathbf{\sin B}} = \frac{\mathbf{c}}{\mathbf{\sin C}}$$

$$\mathbf{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$\mathbf{b^2 = a^2 + c^2 - 2ac\cos B}$$

$$\mathbf{c^2 = a^2 + b^2 - 2ab\cos C}$$

$$\mathbf{\sin 2x = 2\sin x\cos x}$$

$$\mathbf{\cos 2x = \cos^2 x - \sin^2 x}$$

$$\mathbf{\cos 2x = 2\cos^2 x - 1}$$

$$\mathbf{\cos 2x = 1 - 2\sin^2 x}$$

$$\mathbf{\tan 2x = \frac{2\tan x}{1 - \tan^2 x}}$$

$$\mathbf{\sin^2 x + \cos^2 x = 1}$$

$$\mathbf{\sec^2 x = 1 + \tan^2 x}$$

$$\mathbf{\operatorname{cosec}^2 x = 1 + \cot^2 x}$$

$$\mathbf{\sin^2 x = \tfrac{1}{2}(1 - 2\cos 2x)}$$

$$\mathbf{\cos^2 x = \tfrac{1}{2}(1 + 2\cos 2x}$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

DIFFERENTIAL CALCULUS

Standard Derivatives:

y or f(x)	$\frac{dy}{dx}$ or f'(x)	y or f(x)	$\frac{dy}{dx}$ or f'(x)
ax^n	$an\,x^{n-1}$	sin ax	a cos ax
e^{ax}	ae^{ax}	cos ax	-a sin ax
ln ax	$\frac{1}{x}$	tan ax	$a\sec^2 ax$

Product Rule: when y = uv where u & v are functions of x, then: $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Quotient Rule: when y = u/v where u & v are functions of x, then: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain Rule or 'function of a function': if u is a function of x, then: $\frac{dy}{dx} = \frac{dy}{du} x \frac{du}{dx}$

Numerical Differentiation - 3 point equation: $\frac{dy}{dx} = \frac{f(a+h) - f(a-h)}{2h}$

Maximum or minimum values: If $y = f(x)$ then $\frac{dy}{dx} = 0$ for stationary points

To find whether the stationary points are max, min or points of inflection:

Differentiate again to get $\frac{d^2y}{dx^2}$.

If the value is *positive*, the point is a *minimum*
If the value is *negative*, the point is a *maximum*
If the value is *zero at the point*, < 0 on one side and > 0 on the other, the point is a *point of inflection*

Partial Differentiation:Rate of change

If z = f(u, v, ….) and $\frac{du}{dt}, \frac{dv}{dt}$, …denote the rate of change of u, v, ….., then the rate of change of z, $\frac{dz}{dt}$ is given by: $\frac{dz}{dt} = \frac{dz}{du}.\frac{du}{dt} + \frac{dz}{dv}.\frac{dv}{dt} + ...$

Small changes If $z = f(u, v, ...)$ and du, dv…denote small changes in u, v, ….., then the corresponding change in z, dz is given by: $dz \approx \frac{dz}{du}du + \frac{dz}{dv}dv + ...$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

INTEGRAL CALCULUS

Standard Integrals

y	$\int ydx$	y	$\int ydx$
ax^n	$a\frac{x^{n+1}}{n+1}+c$ (except where n = -1)	$\cos ax$	$\frac{1}{a}\sin ax + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$	$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\frac{1}{x}$	$\ln \lvert x\rvert + c$	$\sec^2 ax$	$\frac{1}{a}\tan ax + c$

Integration by parts:

If u and v are both functions of x, then: $\int u\frac{dv}{dx}.dx = uv - \int v\frac{du}{dx}.dx$

Mid-Ordinate Rule: *Area = b x length of the mid-ordinates*

Trapezium Rule: $Area = b\left[\frac{1}{2}(y_1 + y_n) + (y_2 + y_3 + ...)\right]$

Simpson's Rule: $Area = \frac{b}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ...) + 2(y_2 + y_4 + ...)\right]$

Volumes of Solids of Revolution

Volume of a solid object generated when curve $y = f(x)$ is rotated through 360° about the x-axis is $V = \pi\int_{x_1}^{x_2} y^2 dx$

Volume of a solid object generated when curve $x = f(y)$ is rotated through 360° about the y-axis is $V = \pi\int_{y_1}^{y_2} x^2 dx$

STATISTICS

Mean: $\bar{x} = \frac{\Sigma f.x}{\Sigma f}$

Standard Deviation: $\sigma = \sqrt{\frac{\Sigma f.(x-\bar{x})^2}{\Sigma f}}$

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

DIFFERENTIAL EQUATIONS

First Order Differential Equations: If $\frac{dy}{dx} = f(x)$ then $y = \int f(x)$

If $\frac{dy}{dx} = f(y)$ then $\int f dx = \int \frac{dy}{f(y)}$ If $\frac{dQ}{dt} = kQ$ then $Q = Ae^{kt}$ (where A and k are constants)

Integrating Factor Method: If $\frac{dy}{dx} + Py = Q$

i. Rearrange the equation into the form above (if necessary)
ii. Identify P & Q
iii. Find the Integrating Factor $I = e^{\int Pdx}$
iv. Substitute I into: $y.I = \int IQdx$
v. Integrate the RHS (by parts or a relevant substitution) to give the General Solution.
vi. Substitute values to get the Particular Solution

Second Order Differential Equations (Homogenous type):

i. If $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ (where a, b & c are constants)
ii. Rewrite the equation as $(aD^2 + bD + c)y = 0$
iii. Substitute m for D and solve the auxiliary quadratic equation: $am^2 + bm + c = 0$
iv. If the roots are:
   a) ***Real and different***, say m = α and m =β, then the General Solution is:
   $y = Ae^{\alpha x} + Be^{\beta x}$
   b) ***Real and equal,*** say m = α twice, then the General solution is:
   $y = (Ax + B)e^{\alpha x}$
   c) ***Complex,*** say $m = \alpha \pm j\beta$, then the General solution is:

$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

v. Find the Particular solution by substituting values of y, x and dy/dx into the General solution and it's derivative.

Second Order Differential Equations (Non-Homogenous type):

i. If $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ (where a, b & c are constants)
ii. Solve the complementary function y_c (see above)
iii. Use the following trial functions for the particular integral y_p:

If $F(x) = k$ assume $y = C$ If $F(x) = kx$ $y = Cx + D$

If $F(x) = kx^2$ assume $y = Cx^2 + Dx + c$ If $F(x) = Ce^{kx}$ $y = Ce^{kx}$

If $F(x) = C\sin kx$ or $D\cos kx$ $y = C\cos x + D\sin x$