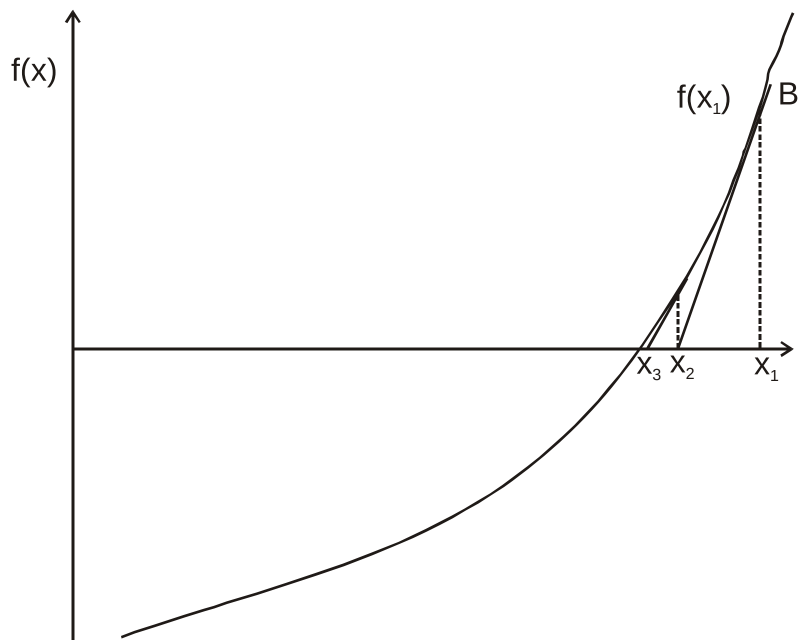
Newton-Raphson Method Handout #8

The Newton-Raphson Method

The *Newton-Raphson* method and its modification is probably the most widely used of all root-finding methods. Starting with an initial guess x_1 at the root, the next guess x_2 is the intersection of the tangent from the point $[x_1, f(x_1)]$ to the x-axis. The next guess x_3 is the intersection of the tangent from the point $[x_2, f(x_2)]$ to the x-axis as shown in Figure 1.6. The process can be repeated until the desired tolerance is attained.



Graphical depiction of the *Newton-Raphson* method.

The *Newton-Raphson* method can be derived from the definition of a slope

$$f'(x_1) = \frac{f(x_1) - 0}{x_1 - x_2} \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, from the point $[x_n, f(x_n)]$, the next guess is calculated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

The derivative or slope $f(x_n)$ can be approximated numerically as

$$f'(x_n) = \frac{f(x_n + \Delta x) - f(x_n)}{\Delta x}$$

Example

Solve $f(x) = x^3 + 4x^2 - 10$ using the the *Newton-Raphson* method for a root in [1, 2].

Solution

From the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$f(x_n) = x_n^3 + 4x_n^2 - 10 \Rightarrow f'(x_n) = 3x_n^2 + 8x_n$$

$$x_{n+1} = x_n - \frac{x_n^3 + 4x_n^2 - 10}{3x_n^2 + 8x_n}$$

Using the initial guess, $x_n = 1.5$, x_{n+1} is estimated as

$$x_{n+1} = 1.5 - \frac{1.5^3 + 4 \times 1.5^2 - 10}{3 \times 1.5^2 + 8 \times 1.5} = 1.3733$$