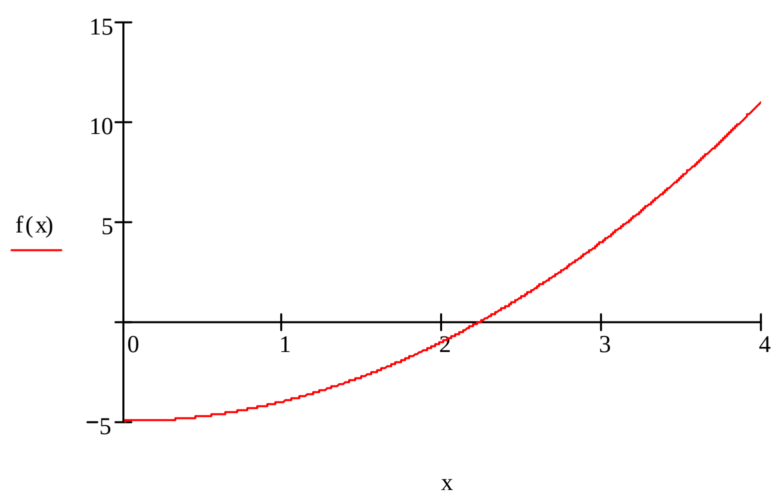
Bisection Method Handout #7

Roots – When you have a function of one variable, f(x), the roots of that function are the values of x which make f(x) = 0

The Bisection Method

Description of Method

Step 1: Choose lower, x_l, and upper, x_u, guesses for the root such that the root changes sign over the interval. This can be checked by ensuring that $f(x_l)\cdot f(x_u) < 0$.



Step2: An estimate of the root, x_r, is determined by

$$x_r = \frac{x_l + x_u}{2}$$

Step 3: Make the following evaluations to determine in which subinterval the root lies:

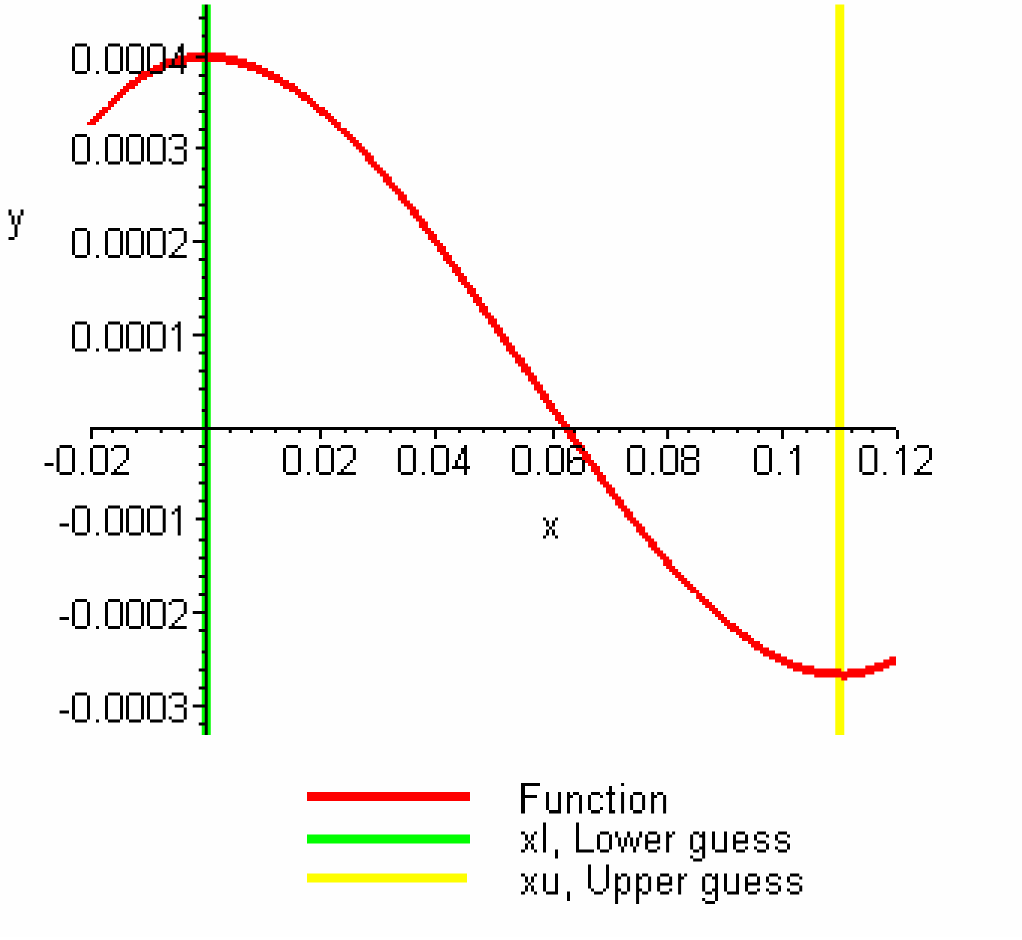
(a) if $f(x_l)\cdot f(x_r) < 0$, the root lies in the lower subinterval. Therefore, set $x_u = x_r$ and return to step 2.

(b) if $f(x_l)\cdot f(x_r) > 0$, the root lies in the lower subinterval. Therefore, set $x_l = x_r$ and return to step 2.

(c) if $f(x_l)\cdot f(x_r) = 0$, the root equals x_r. Terminate the computation.

Example $f(x) = x^3 - 0.165x^2 + 3.993\text{x}10^{-4}$

Entered function on given interval with upper and lower guesses



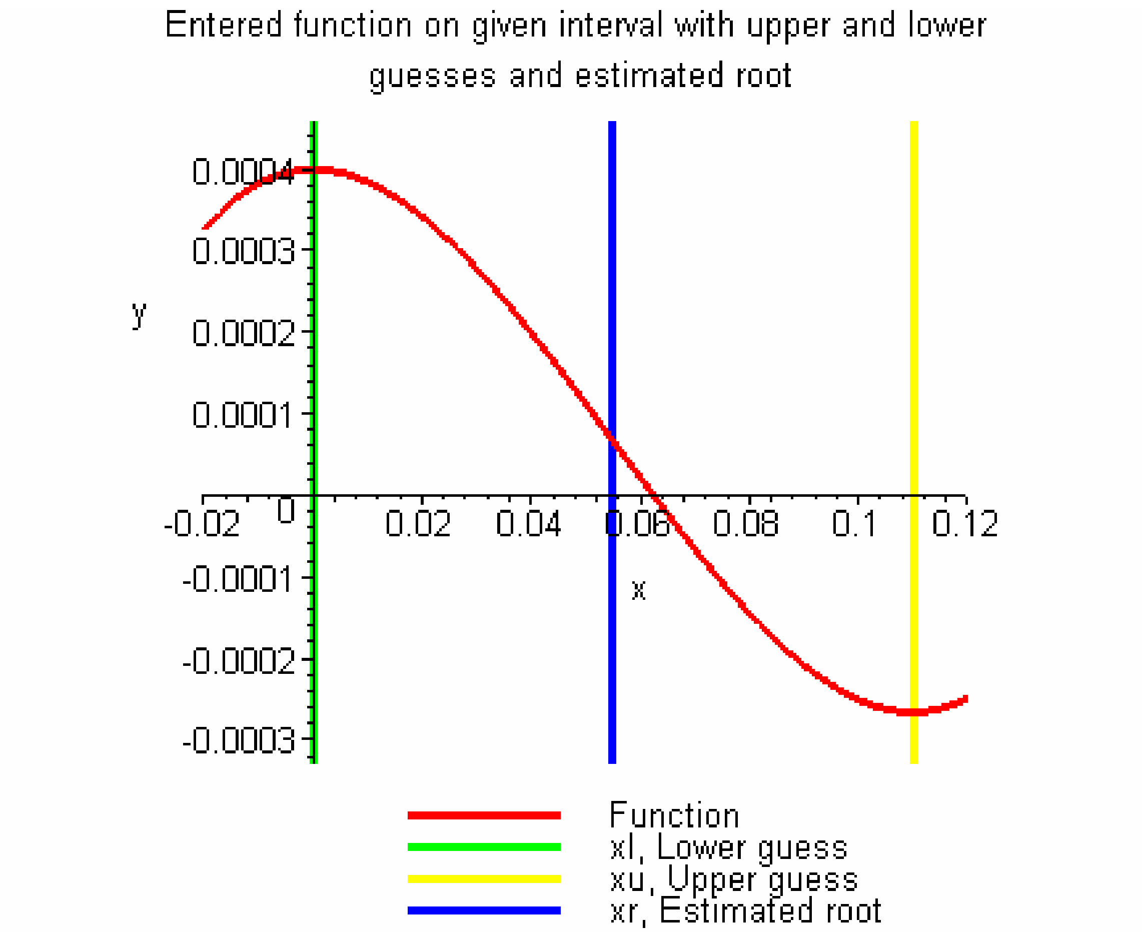
Choose the bracket

$$x_\ell = 0.00$$
$$x_u = 0.11$$

Check the interval validity

$$f(0.0) = 3.993\text{x}10^{-4}$$
$$f(0.11) = -2.662\text{x}10^{-4}$$

Iteration #1



$x_\ell = 0, x_u = 0.11$

$x_m = \dfrac{0+0.11}{2} = 0.055$

$f(0) = 3.993\text{x}10^{-4}$

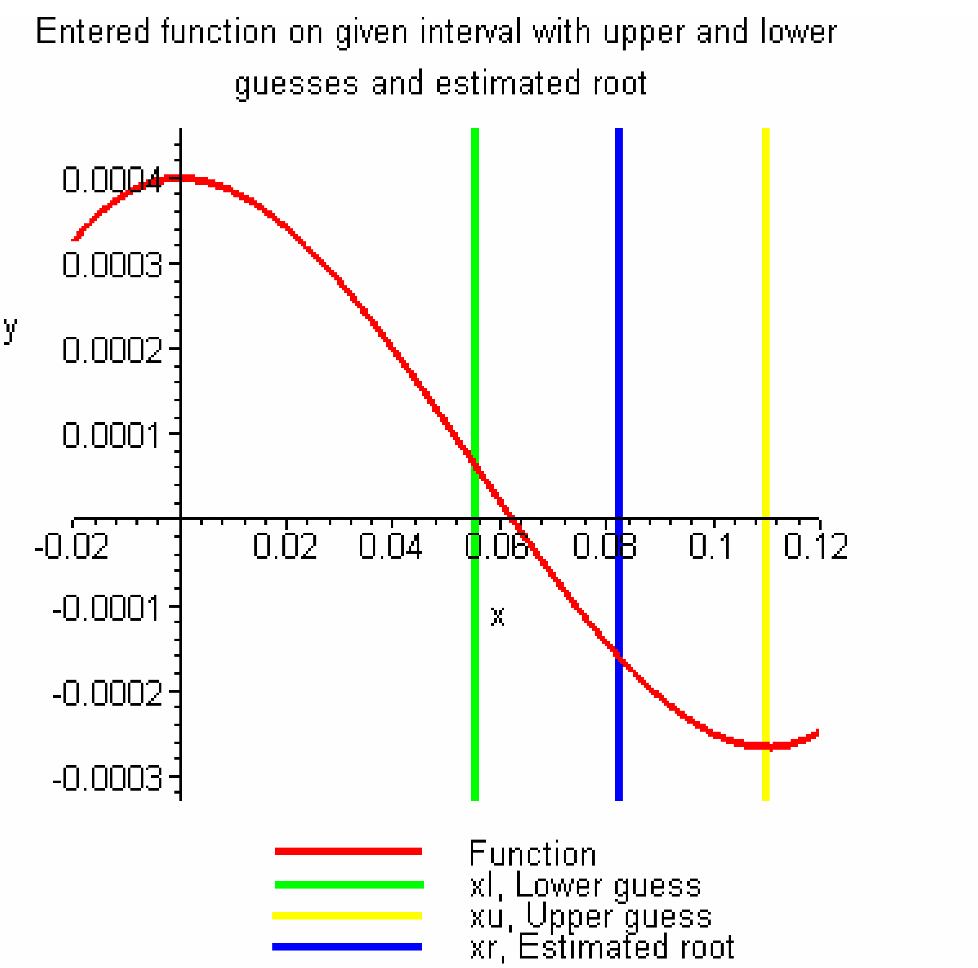
$f(0.11) = -2.662\text{x}10^{-4}$

$f(0.055) = 6.655\text{x}10^{-5}$

$x_\ell = 0.055$

$x_u = 0.11$

Iteration #2



$x_\ell = 0.055, x_u = 0.11$

$x_m = \dfrac{0.055 + 0.11}{2} = 0.0825$

$|\in_a| = 33.33\%$

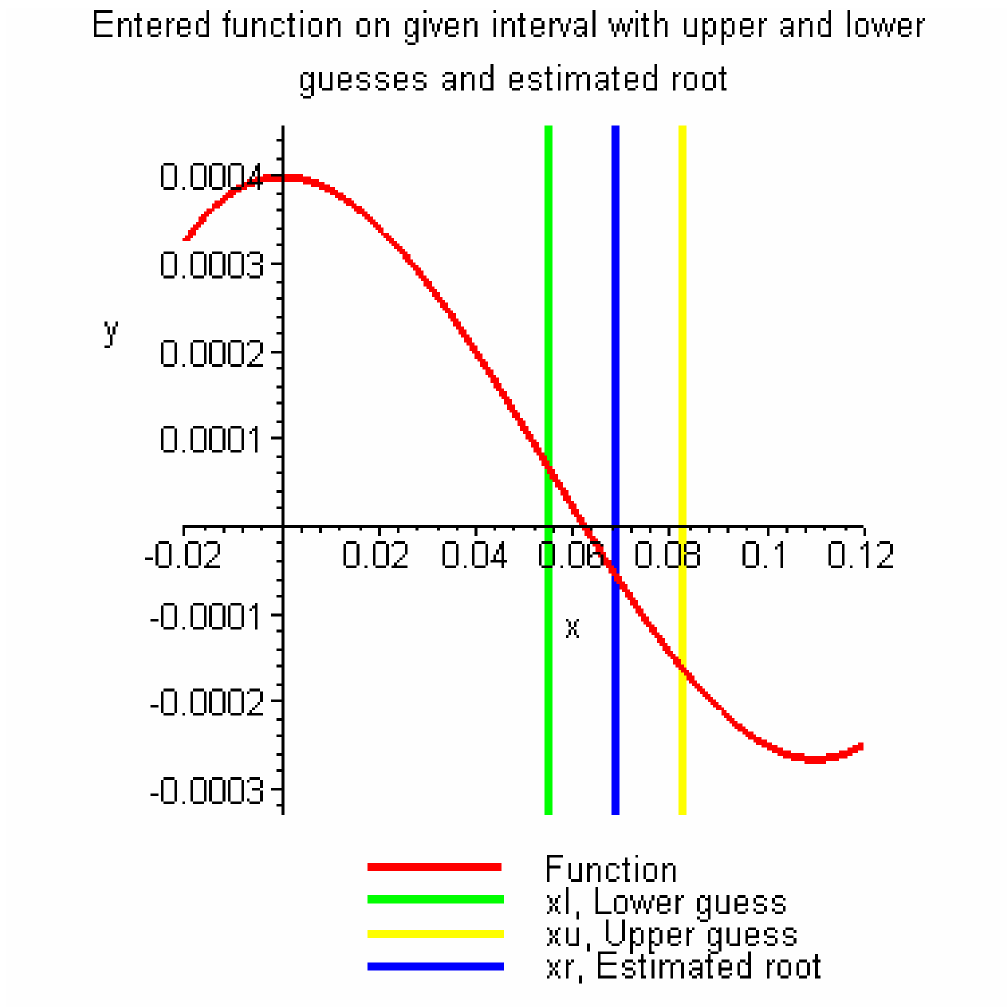
$f(0.055) = 6.655\text{x}10^{-5}$

$f(0.11) = -2.662\text{x}10^{-4}$

$f(0.0825) = -1.62216\text{x}10^{-4}$

$x_\ell = 0.055, x_u = 0.0825$

Iteration #3



$x_\ell = 0.055, x_u = 0.0825$

$$x_m = \frac{0.055 + 0.0825}{2} = 0.06875$$

$|\in_a| = 20\%$

$f(0.055) = 6.655\text{x}10^{-5}$

$f(0.0825) = -1.62216\text{x}10^{-4}$

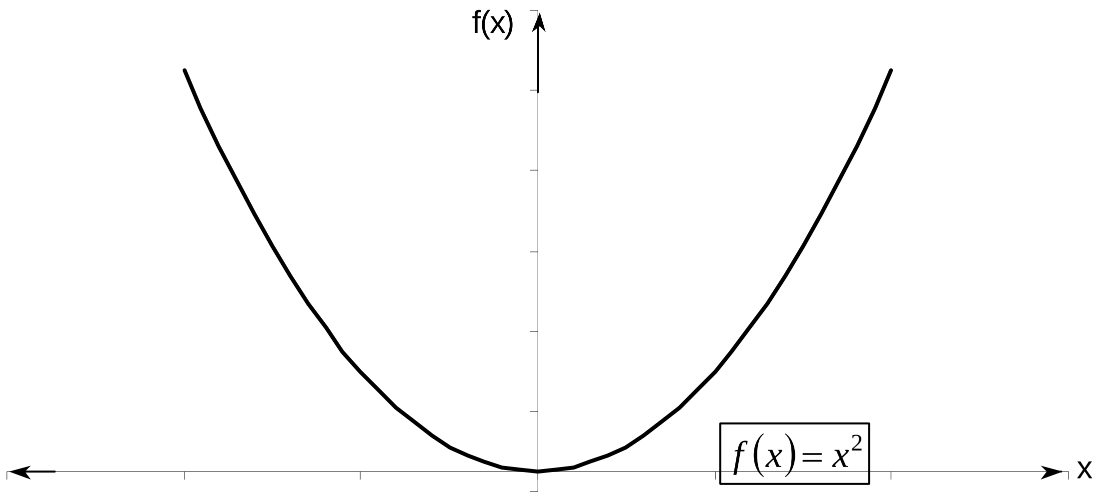
$f(0.06875) = -5.5632\text{x}10^{-5}$

Advantages

- Always convergent
- The root bracket gets halved with each iteration - guaranteed

Disadvantages

- Slow convergence
- If one of the initial guesses is close to the root, the convergence is slower
- If a function f(x) is such that it just touches the x-axis it will be unable to find the lower and upper guesses.



- Function changes sign but root does not exist