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Number Systems Handout #2

Topic	Interpretation

Instead of using ten unique digits to represent numbers, for binary representations we use only two: 0 and 1. The word *bit* is used as an abbreviation for a *bi*nary digi*t*, and thus we will refer to 0 or 1 as bits when we discuss binary numbers.

Binary Integers

To begin, we can consider the process of counting in binary: each time we increment an integer, we add 1 to the least significant bit and if the sum is greater than 1, we increment the next highest bit, possibly going on in this manner:

```
   1
  10
  11
 100
 101
 110
 111
1000
1001
1010
1011
1100
```

Thus, the first number represents the decimal integer 1, the second represents the decimal integer 2, and so on. As well as calling these binary numbers, another term used to describe such numbers is to state that they are in *base two* (or *base 2*). The corresponding values are shown in Table 1.

Table 1. Binary and decimal equivalent numbers.

Binary	Decimal
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9
1010	10
1011	11
1100	12

Distinguishing Binary Numbers

One problem is distinguishing between a binary 10 (= 2)and a decimal 10. When the context is not obvious, we suffix any decimal integer with a subscript 10 (to represent base 10) and any binary number with a subscript 2 (to represent base 2). For example, 10_{10} = 1010_2 and 5632_{10} = 1011000000000_2.

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Binary Number Representation

We use two bits, 0 and 1, to represent numbers. An (n + 1)-bit binary integer is a number of the form $b_n \cdots b_2 b_1 b_0$ where each b_i is a bit and $b_n = 1$. This represents the number $\sum_{i=0}^{n} b_i 2^i$

When a bit $b_i = 1$ is incremented, it is replaced by 0 and the bit b_{i+1} is incremented by 1.

Conversion from Binary to Decimal

From the definition, we may simply calculate the sum. For example, the integer 11010_2 represents $1 \cdot 16 + 1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 0 \cdot 1 = 16 + 8 + 4 = 28$. The real number 1.011 represents $1 \cdot 1 + 0 \cdot 0.5 + 1 \cdot 0.25 + 1 \cdot 0.125 = 1 + 0.25 + 0.125 = 1.375$.

Conversion from Decimal to Binary

This is a little more tedious and there is nothing insightful to be gained from memorizing such an algorithm. If it is a decimal integer, keep dividing by 2 and keep track the remainders. If it is a real number x, find the largest power n of 2 such that $2^{-n}x < 1$ and keep track of whether, by multiplying successively by two whether or not the integer part is 0 or 1 and discarding the integer part with each step

Example 1

Add the binary integers 101101_2 and 10101_2.

```
 1111 1         carry
  101101
+  10101
 -------
 1000010
```

Example 2

Multiply the binary integers 1011_2 and 101_2.

```
     1011
   x  101
     ----
    1          carry for sum
     1011
    0000
 + 1011
   ------
   110111
```

Note that there cannot be any carries in the multiplication: you either have 0 × 1011 or 1 × 1011, a much simpler rule than that for decimal multiplication.

Example 3

Convert the binary number 10011.01101_2 from binary to decimal.

This number represents:

$1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} + 1 \cdot 2^{-5}$ which equals
$16 + 2 + 1 + 0.25 + 0.125 + 0.03125 = 19.40625$.