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Error Arithmetic Handout #1

Topic	Interpretation
Given any mathematical expression, it follows that if the variables have error, then the result will have an associated error, as well. This passing of error from variables to the result is termed *error propogation*. It is possible for computations to magnify this error, however, the amount the error is maginfied depends on the operation:	Example 1 suppose the two values 3.55 and 3.54 represent numbers in the ranges 3.545 to 3.555 and 3.535 to 3.545, respectively. If we calculate 3.55 - 3.54 = 0.01, the actual value could be anywhere in the range 0 to 0.02. Thus while we are reporting that the difference is positive, the actual difference may be zero (or, if the error widths were slightly larger, the difference may be negative).
Precision and accuracy Two words we will be using to describe how good a measurement or approximation is to an actual value are *precision* and *accuracy*. **Absolute and Relative errors** There are two techniques for measuring error: the absolute error of an approximation and the relative error of the approximation. The first gives how large the error is, while the second gives how large the error is relative to the correct value. **Calculating Absolute Error** Given an approximation a of a value x, the absolute error E_{abs} is calculated using the formula: $$E_{abs} = \| x - a \|$$ **Calculating Relative Error** Given an approximation a of a value x, the relative error E_{rel} is calculated using the formula: $$E_{rel} = \frac{\|x - a\|}{\|x\|}$$	Example 2 What are the absolute and relative errors of the approximation 3.14 to the value π? $E_{abs} = \|3.14 - \pi\| \approx 0.0016$ $E_{rel} = \|3.14 - \pi\|/\|\pi\| \approx 0.00051$ Example 3 A resistor labeled as 240 Ω is actually 243.32753 Ω. What are the absolute and relative errors of the labeled value? $E_{abs} = \|240 - 243.32753\| \approx 3.3\ \Omega$ $E_{rel} = \|240 - 243.32753\|/\|243.32753\| \approx 0.014$Note: the label is the approximation of the actual value. Example 4 The voltage in a high-voltage transmission line is stated to be 2.4 MV while the actual voltage may range from 2.1 MV to 2.7 MV. What is the maximum absolute and relative error of voltage? $E_{abs} = \|2.4 - 2.1\| = 0.3$ MV $E_{rel} = \|2.4 - 2.1\|/\|2.1\| \approx 0.14$ $E_{abs} = \|2.4 - 2.7\| = 0.3$ MV $E_{rel} = \|2.4 - 2.7\|/\|2.7\| \approx 0.11$ Thus, the maximum absolute error is 0.3 MV but the maximum relative error is 0.14.Note: as before, the stated voltage is an approximation of the actual voltage.

Significant figures

Given a relative error E_{rel}, find the largest integer n such that

$E_{rel} < 0.5\ 10^{-n}$.

If the relative error is greater than 0.5, state that the approximation does not have any significant digits.

In general, the number of significant digits between a number and its approximation are equal to the number of leading digits which are equal, though this is only a rule of thumb, and if the most significant digit is 1 or 2, it is most useful to ignore it when counting the number of significant digits.

Example 5

What is the number of significant digits of the approximation 3.14 to the value π?

$E_{rel} = |3.14 - \pi|/|\pi| \approx 0.00051 \le 0.005 = 0.5 \cdot 10^{-2}$, and therefore it is correct to two significant digits.

This example demonstrates a weakness in the concept of significant digits: in this example, it would be almost better to say that 3.14 approximates π to *almost* or *approximately* three significant digits.

Example 6

What is the number of significant digits of the label 240 Ω when the correct value is 243.32753 Ω?

$E_{rel} = |240 - 243.32753|/|243.32753| \approx 0.014 \le 0.05 = 0.5 \cdot 10^{-1}$, and therefore it is correct to one significant digit.

Example 7

To how many significant digits is the approximation 1.998532 when the actual value is 2.001959?

$E_{rel} = |1.998532 - 2.001959|/|2.001959| \approx 0.0017 \le 0.005 = 0.5 \cdot 10^{-2}$ and therefore it is correct to two digits.