معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

Series
Tutoring Sheet #5 – Solution

1. An arithmetic progression has fifth term equal to 4 ,and the sum of its first 13 terms is 65.Find the first term and the common difference.

$a_5 = a + 4d = 4$

$S_{13} = (13/2)[2a + (13-1)d] = 65 \Rightarrow 13a + 78d = 65$

Solving the above two equations simultaneously for a and d :

$a = 2$, $d = ½$

2. Find an arithmetic series(first term and common difference) where the fourth term is 5 and the sum of the third and the eighth terms is 1.Then find the 15^{th} term.

$a_4 = a + 3d = 5$

$a_3 + a_8 = a + 2d + a + 7d = 1 \Rightarrow 2a + 9d = 1$

Solving the above two equations simultaneously for a and d :

$a = 14$, $d = -3$

3. Find three consecutive terms of a geometric sequence such that their product is 64 and their sum is 21.

[Hint: assume the terms : a/r , a , ar)

$(a/r)(a)(ar) = 64 \Rightarrow a^3 = 64 \Rightarrow a = 4$

$a/r + a + ar = 21 \Rightarrow 4/r + 4 + 4r = 21$

$\Rightarrow 4 + 4r + 4r^2 = 21r \Rightarrow 4r^2 - 17r + 4 = 0$

$\Rightarrow r = 4$ or $r = ¼$

4. In the geometric sequence : 81,27,9, …. Which term is 1/243

$a = 81$, $r = 27/81 = 1/3$

$1/243 = ar^{n-1} = 81(1/3)^{n-1} \Rightarrow (1/3)^{n-1} = 1/(243)(81)$

$3^{-n+1} = (243)(81) \Rightarrow 3^{-n+1} = 3^5 . 3^3 = 3^8 \Rightarrow -n + 1 = 9$

$\Rightarrow n = 8$

5. Find a geometric sequence where the third term exceeds the second by 6 and the fourth term exceeds the third by 4.

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$a_3 = a_2 + 6 \Rightarrow ar^2 = ar + 6 \Rightarrow a = 6/(r^2 - r)$
$a_4 = a_3 + 4 \Rightarrow ar^3 = ar^2 + 4 \Rightarrow a = 4/(r^3 - r)$
$6/(r^2 - r) = 4/(r^3 - r) \Rightarrow 6r^3 - 6r = 4r^2 - 4r \Rightarrow 3r^3 - 2r^2 - r = 0$
$\Rightarrow r(r^2 - r - 1) = 0 \Rightarrow r = 0$ trivial or $r^2 - r - 1 = 0$

$$\Rightarrow r = \frac{1 \pm \sqrt{5}}{2}$$

6. A geometric progression has second term equal to 2 and a sum to infinity of 9.Show that there are two possible values of the common ratio and find these.
$a_2 = 2 \Rightarrow ar = 2 \Rightarrow a = 2/r$

$$\frac{a}{1-r} = 9 \Rightarrow 9 - 9r = a \Rightarrow 9 - 9r = 2/r \Rightarrow -9r^2 + 9r - 2 = 0$$

$\Rightarrow r = 1/3$ or $r = 2/3$

7. An arithmetic progression has first term equal 3 and the sixth term is double the third .Find the sum of the first 9 terms.
$a = 3$; $a_6 = 2a_3 \Rightarrow a + 5d = 2(a + 2d) \Rightarrow a = 3d$
$\Rightarrow 3 = 3d \Rightarrow d = 1$
$S = (9/2)[2a + (n-1)d] = (9/2)[2(3) + 8(1)] = 63$

8. The sum of first n terms of an arithmetic progression is :
$S_n = n^2 - 3n$ .Find the fourth term and the n^{th} term.
$S_1 = a = 1^2 - 3(1) = -2$,$S_2 = 2^2 - 3(2) = -2$
$S_2 = a + a_2 \Rightarrow a_2 = S_2 - S_1 = -2 + 2 = 0$
$a_n = S_n - S_{n-1} = n^2 - 3n - (n-1)^2 + 3(n-1) = 2n - 4$

9. How many terms are needed of the arithmetic progression 1,3,5,….. to get a sum of 1521.
$S = (n/2)[2a + (n-1)d] = 1521$
$\Rightarrow (n/2)[2 + (n-1)(2)] = 1521 \Rightarrow n^2 = 1521 \Rightarrow n = 39$

10. Find the sum of the first 21 terms of the arithmetic progression:
ln 10 , ln 20 , ln 40 ,………
$a = \ln 10$; $d = \ln 20 - \ln 10 = \ln(20/10) = \ln 2$
$S = (21/2)[2(\ln 10) + (21-1)(\ln 2)] = 21\ln 10 + 210 \ln 2$