Partial Fractions
Tutoring Sheet #3 – Solution

1. $\dfrac{3}{2(x-6)} - \dfrac{3}{2(x-2)}$

2. $\dfrac{1}{x+1} + \dfrac{3}{x-3}$,

3. $\dfrac{5}{7(x+2)} + \dfrac{16}{7(x-5)}$

4. $\dfrac{-1}{8(x+1)} + \dfrac{5}{24(x-3)} - \dfrac{1}{12(x+3)}$

5. $\dfrac{1}{5(x-2)} + \dfrac{(-x-2)}{5(x^2+1)}$

6. $\dfrac{5x+2}{9(x^2+2)} - \dfrac{10}{9(2x+1)}$,

7. $\dfrac{-8}{9(x+2)} - \dfrac{1}{3(x+2)^2} + \dfrac{16}{9(2x+1)}$

8. $\dfrac{1}{x^2-7x+10} = \dfrac{1}{(x-2)(x-5)} = \dfrac{A}{x-2} + \dfrac{B}{x-5}$

        1= A(x-5) + B(x-2)
        Choose x = 2 , 1 = -3A , A= -1/3
        Choose x = 5 , 1 = 3B , B= 1/3

$\dfrac{1}{x^2-7x+10} = \dfrac{1}{(x-2)(x-5)} = \dfrac{A}{x-2} + \dfrac{B}{x-5} = \dfrac{-1/3}{x-2} + \dfrac{1/3}{x-5}$

9. $\dfrac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} = x + 1 + \dfrac{4x}{x^3 - x^2 - x + 1}$

Note: long division since degree of numerator is greater than degree of denominator.

$\dfrac{4x}{(x-1)^2 (x+1)}$

Note: Factor $x^3 - x^2 - x + 1 = x^2(x-1) - (x-1) = (x-1)(x^2 - 1) = (x-1)(x-1)(x+1)$

$\dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$

Note: Partial fraction decomposition since $(x-1)^2$'s factor is linear. There is a constant on

top for the and power and first power

$4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

Note: multiply by Least common denominator $(x-1)^2$ $(x+1)$

$4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

Choose $x = 1$, $4 = 0 + 2B + 0$, $4 = 2B$, $B = 2$

Choose $x = -1$, $-4 = 0 + 0 + 4C$, $4C = -4$, $C = -1$

Choose $x = 0$, $0 = -A + B + C$, $-A + 2 - 1 = 0$, $A = 1$

$A = 1$, $B = 2$, $C = -1$

$\dfrac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} = x + 1 + \dfrac{4x}{x^3 - x^2 - x + 1} = x + 1 + \dfrac{1}{x-1} + \dfrac{2}{(x-1)^2} - \dfrac{1}{x+1}$