Sharjah Institute of Technology

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Factor and Remainder Theorems

Tutoring Sheet #2 – Solution

1. Find the remainder of division in each of the following

a. $x^3 - 3x + 7$ by $x - 3$
$R = f(3) = 3^3 - 3(3) + 7 = 25$

b. $x^3 - 2x^2 - 9x + 18$ by $x + 3$

$R = f(-3) = (-3)^3 - 2(-3)^2 - 9(-3) + 18 = 0$

Note : x + 3 is a factor.

c. $x^2 + 3$ by $2x - 3$

$R = f(3/2) = (3/2)^2 + 3 = 21/4$

2. Find the remainder R by long division and by the remainder theorem: $(2x^4 - 10x^2 + 30x - 60) \div (x + 4)$

Remainder $= f(-4) = 2(-4)^4 - 10(-4)^2 + 30(-4) - 60 = 172$

$$
\begin{array}{r|l}
 & 2x^3 - 8x^2 + 22x - 58 \\
\hline
x+4 & 2x^4 + 0x^3 - 10x^2 + 30x - 60 \\
 & \underline{2x^4 + 8x^3} \\
 & -8x^3 - 10x^2 \\
 & \underline{-8x^3 - 32x^2} \\
 & 22x^2 + 30x \\
 & \underline{22x^2 + 88x} \\
 & -58x - 60 \\
 & \underline{-58x - 232} \\
 & 172
\end{array}
$$

3. Use the factor theorem to decide if (x - 2) is a factor of

$f(x) = 2x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8.$

$f(x) = x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8$

$f(2) = (2)^5 - 2(2)^4 + 3(2)^3 - 6(2)^2 - 4(2) + 8 = 0$

Since f(2) = 0, we can conclude that (x - 2) is a factor.

4. Let $f(x) = x^3 - 7x + 6$. Solve the equation $f(x) = 0$

$f(1) = 1^3 - 7 + 6 = 0 \Rightarrow x = 1$ is a root and $(x - 1)$ is a factor.

To find the other factor we divide f(x) by (x – 1)

$$\begin{array}{c|cccc} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

Quotient = $x^2 + x - 6$ and remainder = 0

So, $f(x) = (x - 1)(x + 3)(x - 2)$. For $f(x) = 0$, $x = -3, 1, 2$.

3. $f(x) = 4x^3 + 3x - 18 = 0$

Possible rational roots are $\pm\dfrac{1, 2, 3, 6, 9, 18}{1, 2, 4}$

f(3/2) = $4(3/2)^3$ + 3(3/2) – 18 = 27/2 + 9/2 – 18 = 36/2 – 18
= 18 – 18 = 0

therefore $x = \dfrac{3}{2}$ is a root. So $2x - 3$ is a factor.

$$\begin{array}{rl}
 & 2x^2 + 3x + 6 \\
2x-3 \,\big) & 4x^3 + 0x^2 + 3x - 18 \\
 & \underline{-(4x^3 - 6x^2)} \\
 & \quad 6x^2 + 3x - 18 \\
 & \quad \underline{-(6x^2 - 9x)} \\
 & \qquad 12x - 18 \\
 & \qquad \underline{-(12x - 18)} \\
 & \qquad\qquad 0
\end{array}$$

So $f(x) = 4x^3 + 3x - 18 = (2x - 3)(2x^2 + 3x + 6)$

The quadratic quotient will not factor. Using the quadratic formula we find that the three roots are

$$x = \frac{3}{2}, \quad \frac{-3 \pm i\sqrt{39}}{4}.$$