Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

Integration III
Tutoring Sheet #17 – Solution

1.Show that : $\dfrac{t^2+4t+4}{t+3} = t+1+\dfrac{1}{t+3}$

The right hand side: $t+1+\dfrac{1}{t+3} = \dfrac{(t+1)(t+3)}{t+3} = \dfrac{t^2+4t+4}{t+3}$

Hence find: $\displaystyle\int_1^2 \frac{t^2+4t+4}{t+3}\,\text{dt} = \int_1^2 \left(t+1+\frac{1}{t+3}\right)\text{dt}$

$$= \left.\frac{t^2}{2}+t+\ln(t+3)\right|_1^2 = \frac{2^2}{2}+2+\ln(2+3)-\left(\frac{1^2}{2}+1+\ln(1+3)\right)$$

$$= 4 + \ln 5 - \frac{3}{2} - \ln 4 = \frac{5}{2}+\ln 5-\ln 4$$

2. Determine $\displaystyle\int \frac{x-3}{x^2-6x+5}\,dx$

Let u = x^2 -6x + 5 ⇒ du = (2x-6)dx=2(x-3)dx

⇒ (x-3)dx = $\dfrac{du}{2}$:

$$\int \frac{x-3}{x^2-6x+5}\,dx = \int \frac{\frac{du}{2}}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln(x^2-6x+5)+C$$

3. Evaluate the following integrals:

a. $\int \frac{x}{x^2 - x - 2} dx$ by partial fractions:

$$\frac{x}{x^2 - x - 2} = \frac{a}{x+1} + \frac{b}{x-2}$$

$\Rightarrow$ x =a(x-2) + b(x+1)

Choose x = 2 $\Rightarrow$ 2 = a(0) + b(3) $\Rightarrow$ b = $\frac{2}{3}$

Choose x = -1$\Rightarrow$-1 = a(-3) + b(0) $\Rightarrow$ a = $\frac{-1}{3}$

$$\int \frac{x}{x^2 - x - 2} dx = \int \left(\frac{a}{x+1} + \frac{b}{x-2}\right) dx = \int \left(\frac{\frac{-1}{3}}{x+1} + \frac{\frac{2}{3}}{x-2}\right) dx$$

$$= \frac{-1}{3}\ln(x+1) + \frac{2}{3}\ln(x-2) + C$$

b. $\int \frac{dx}{x^2 + 4x + 3}$ Using Integration by partial fractions:

$$\frac{1}{x^2 + 4x + 3} = \frac{a}{x+1} + \frac{b}{x+3}$$

$\Rightarrow$ 1 = a(x+3) + b(x+1)

Choose x = -3 $\Rightarrow$ 1 = a(0) + b(-2) $\Rightarrow$ b = $\frac{-1}{2}$

Choose x = -1 $\Rightarrow$ 1 = a(4) + b(0) $\Rightarrow$ a = $\frac{1}{4}$

$$\int \frac{dx}{x^2+4x+3} = \int \left(\frac{\frac{1}{4}}{x+1} + \frac{\frac{-1}{2}}{x+3} \right) dx = \frac{1}{4}\ln(x+1) \ \frac{-1}{2}\ln(x+3) + C$$

c. $\int x^2 e^x dx$ Using integration by parts:

$u = x^2 \Rightarrow du = 2x\,dx$; $dv = e^x\,dx \Rightarrow v = e^x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 e^x dx = x^2 e^x - \int e^x (2x)\,dx = x^2 e^x - 2\int xe^x\,dx$$

Now $\int xe^x\,dx$ by parts again:

$u = x \Rightarrow du = dx$; $dv = e^x\,dx \Rightarrow v = e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x$$

$$\int x^2 e^x dx = x^2 e^x - 2(xe^x - e^x) + C = (x^2-2x+2)e^x + C$$

d. $\int t \ln t\,dt$ Using integration by parts:

$u = \ln t \Rightarrow du = \frac{1}{t}dt$; $dv = t\,dt \Rightarrow v = \frac{t^2}{2}$

$$\int u\,dv = uv - \int v\,du$$

$$\int t \ln t\,dt = \frac{t^2}{2}\ln t - \int \frac{t^2}{2}\frac{1}{t}\,dt = \frac{t^2}{2}\ln t - \frac{1}{2}\int t\,dt$$

$$= \frac{t^2}{2}\ln t - \frac{1}{2}\frac{t^2}{2} + C = \frac{t^2}{2}\ln t - \frac{t^2}{4} + C$$

e. $\int \frac{2x-1}{x^2-x+3}\,dx$

Note that 2x – 1 is the derivative of x^2 - x + 3

Let u = x^2 - x + 3 ⇒ du = (2x - 1)dx ;substituting:

$$\int \frac{du}{u} = \ln u + C = \ln(x^2 - x + 3) + C$$

f. $\int \frac{\ln x}{x^2}\,dx$

View answer at :

http://www.mathyards.com/vb/showthread.php?&threadid=597

g. $\int x^2\sqrt{x+3}\,dx$

View answer at :

http://www.mathyards.com/vb/showthread.php?&threadid=597

h. $\int x\sin x\,dx$ Using integration by parts:

u = x ⇒ du = dx ; dv = sinx dx ⇒ v = $\int \sin x dx$
⇒ v = - cosx

$\int u\,dv = uv - \int v\,du$

$\int x\sin x\,dx$ = -x cosx - $\int -\cos x\,dx$ = -x cosx + $\int \cos x dx$

= -x cos + sinx + C