



## Integration II

### Tutoring Sheet #16 – Solution

$$1. \int (x^3 - 14x^2 + 20x + 3) dx = \frac{x^4}{4} - \frac{14x^3}{3} + \frac{20x^2}{2} + 3x + C$$

$$2. \int (2 + e^{-x}) dx = 2x - e^{-x} + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$3. \int \left( \frac{1}{t^2} - \frac{2}{\sqrt{t}} \right) dt = \int (t^{-2} - 2t^{-\frac{1}{2}}) dt = \frac{t^{-2+1}}{-2+1} - 2 \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$
$$= \frac{t^{-1}}{-1} - 2 \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{-1}{t} - 4t^{\frac{1}{2}} + C$$

$$4. \int_0^T qe^{qt} dt = q \int_0^T e^{qt} dt = q \times \frac{1}{q} e^{qt} \Big|_0^T = e^{qT} - e^0 = e^{qT} - 1$$

$$5. \int (6x^{-3} + 4x^{-1}) dx = 6 \frac{x^{-3+1}}{-3+1} + 4 \ln x + C = \frac{6x^{-2}}{-2} + 4 \ln x + C$$

$$= -3x^{-2} + 4 \ln x + C$$

$$\int \frac{dx}{x} = \int x^{-1} dx = \ln x + C$$

$$6. \int \left( \frac{3}{x} + 4e^{-5x} \right) dx = 3 \ln x + \frac{4}{-5} e^{-5x} + C = 3 \ln x - \frac{4}{5} e^{-5x} + C$$



$$7. \int \frac{dt}{1-2t} = \frac{-1}{2} \ln(1-2t) + C$$

$$\int \frac{dx}{b+ax} = \frac{1}{a} \ln(b+ax) + C$$

$$8. \int \frac{7}{3-4t} dt = 7 \times \frac{1}{-4} \ln(2-4t) + C = \frac{-7}{4} \ln(2-4t) + C$$

$$9. \int 8xe^{3x^2} dx \quad \text{Let } u = x^2 \Rightarrow du = 2xdx \Rightarrow xdx = \frac{du}{2}$$
$$= \int 8e^{3u} \frac{du}{2} = 4 \int e^{3u} du = 4 \times \frac{1}{3} e^{3u} + C = \frac{4}{3} e^{3x^2} + C$$

$$10. \int \frac{3x^2+2}{x^3+2x-5} dx$$

Note that  $3x^2 + 2$  is the derivative of  $x^3 + 2x - 5$

Let  $u = x^3 + 2x - 5 \Rightarrow du = (3x^2 + 2)dx$  ; substituting:

$$\int \frac{du}{u} = \ln u + C = \ln(x^3 + 2x - 5) + C$$

$$11. \int \sqrt{1-x} dx \quad \text{Let } u = 1-x \Rightarrow du = -dx \Rightarrow dx = -du$$

$$\int \sqrt{u}(-du) = - \int \sqrt{u} du = - \int u^{\frac{1}{2}} du = - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$



$$= \frac{-2}{3} u^{\frac{3}{2}} + C = \frac{-2}{3} (\sqrt{1-x})^{\frac{3}{2}} + C$$

12.  $\int x^2(x^3+1)^4 dx$

Let  $u = x^3 + 1 \Rightarrow du = 3x^2 dx \Rightarrow x^2 dx = \frac{du}{3}$

$$\int u^4 \frac{du}{3} = \frac{1}{3} \int u^4 du = \frac{1}{3} \frac{u^5}{5} + C = \frac{u^5}{15} + C = \frac{(x^3+1)^5}{15} + C$$

13.  $\int \frac{e^{2t}}{e^{2t}+1} dt$

Let  $u = e^{2t} + 1 \Rightarrow du = 2e^{2t} dt \Rightarrow e^{2t} dt = \frac{du}{2}$

$$\int \frac{\frac{1}{2} du}{u} = \frac{1}{2} \ln u + C = \frac{1}{2} \ln(e^{2t} + 1) + C$$

14.  $\int_1^e \frac{3}{x} dx = 3 \ln x \Big|_1^e = 3 \ln e - 3 \ln 1 = 3(1) - 3(0) = 3 - 0 = 3$

$\ln e = 1$ and $\ln 1 = 0$
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