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IntegrationI
Tutoring Sheet #15 – Solution

Evaluate the following integrals:

1. $\int (3x^2 + 2x - 5)dx = 3\frac{x^3}{3} + 2\frac{x^2}{2} - 5x + C = x^3 + x^2 - 5x + C$

2. $\int x\sqrt{x}dx = \int x^1 \times x^{\frac{1}{2}}dx = \int x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2x^{\frac{5}{2}}}{5} + C$

3. $\int_0^4 \frac{dx}{\sqrt{x}} = \int_0^4 \frac{dx}{x^{\frac{1}{2}}} = \int_0^4 x^{\frac{-1}{2}}dx = \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x} = F(4) - F(0)$

$= 2\sqrt{4} - 2\sqrt{0} = 2(2) - 0 = 4$

4. $\int_0^2 x(x^4 + 2)dx = \int_0^2 (x^5 + 2x)dx = \frac{x^6}{6} + 2\frac{x^2}{2} = \frac{x^6}{6} + x^2 = F(2) - F(0)$

$= \left(\frac{2^6}{6} + 2^2\right) - \left(\frac{0^6}{6} + 0^2\right) = \frac{64}{6} - 0 = \frac{32}{3}$

5. $\int \frac{dx}{x^2} = \int x^{-2}dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$

6. $\int \frac{(x+1)dx}{x^2} = \int (x+1)x^{-2}dx = \int (x^{-1}+x^{-2})dx$ = Ln|x| + $\frac{x^{-1}}{-1}$ + C

= Ln|x| - $\frac{1}{x}$ + C

7. $\int \frac{x^3-3x^2+6x}{x}dx = \int (x^3-3x^2+6x)x^{-1}dx = \int (x^2-3x^1+6x^0)\,dx$

= $\frac{x^3}{3}-\frac{3x^2}{2}+6x$ + C

8. $\int (x+2)^9\,dx$; Let u = x + 2 $\Rightarrow$ du = dx

$=\int u^9 du = \frac{u^{10}}{10}$ + C = $\frac{(x+2)^{10}}{10}$ + C

9. $\int_1^2 2x(x^2+3)^4 dx$; Let u = x^2 + 3 $\Rightarrow$ du = 2x dx

= $\int_1^2 u^4 du = \frac{u^5}{5} = \frac{(x^2+3)^5}{5}$ = F(2) – F(1)

= $\frac{(2^2+3)^5}{5}-\frac{(1^2+3)^5}{5} = \frac{7^5}{5}-\frac{4^5}{5}=\frac{7^5-4^5}{5}$

10. $\int x(3x+5)^7\,dx$; Let u = 3x + 5 $\Rightarrow$ du = 3 dx $\Rightarrow$ dx = $\frac{du}{3}$

and $x=\frac{u-5}{3}$;Therefore $\int x(3x+5)^7\,dx = \int \frac{u-5}{3}\times u^7 \times \frac{du}{3}$

$=\frac{1}{9}\int \left(u^8-5u^7\right)du = \frac{1}{9}\left(\frac{u^9}{9}-\frac{5u^8}{3}\right)$ + C = $\frac{u^9}{81}-\frac{5u^8}{27}$ + C

= $\frac{(3x+5)^9}{81}-\frac{5(3x+5)^8}{27}$ + C

11. $\int x\,(2x^2+7)^8\,dx$

Let u = $2x^2$ + 7 $\Rightarrow$ du = 4x dx $\Rightarrow$ x dx = $\frac{du}{4}$

= $\int u^8 \frac{du}{4}$ = $\frac{1}{4}\frac{u^9}{9}$ + C = $\frac{u^9}{36}$ = $\frac{(2x^2+7)^9}{36}$ + C

12. $\int x\sqrt{x-1}\,dx$

Let u = $\sqrt{x-1}$ $\Rightarrow$ u^2 = x – 1 $\Rightarrow$ x = u^2 + 1
dx = 2u du

= $\int (u^2+1)(u)(2u)du$ = $\int (u^2+1)(2u^2)du$ = $2\int (u^2+1)(u^2)du$

= $2\int (u^4+u^2)du$ = $2\frac{u^5}{5}$ + $2\frac{u^3}{3}$ + C

= $\frac{2(\sqrt{x-1})^5}{5}$ + $\frac{2(\sqrt{x-1})^3}{3}$ + C