



## Series

## Handout #5

Topic	Interpretation
<p><b>Arithmetic Sequence (Progression)</b>  <math>a, a+d, a+2d, a+3d, \dots, a+(n-1)d</math>  <b><math>n^{\text{th}}</math> term : <math>a_n = a + (n-1)d</math></b>  <b>Sum of first n terms:</b></p> $S_n = \frac{n}{2}(a + a_n)$ $= \frac{n}{2} \{ 2a + (n-1)d \}$ <p><u>Example 3:</u>                      Find the sum of the first n odd positive integers:  <math>1+3+5+7+\dots+</math></p> <p><b>Geometric Sequence (Progression)</b>  <math>a, ar, ar^2, ar^3, \dots, ar^{n-1}</math>  <b><math>n^{\text{th}}</math> term : <math>a_n = ar^{n-1}</math></b>  <b>Sum of first n terms:</b></p> $S_n = a \times \frac{r^n - 1}{r - 1}$ <p><b>Sum to infinity of a G.P.</b>                      A geometric sequence is said to be infinite when <math>-1 &lt; r &lt; 1</math>                      In this case :</p> $S_{\infty} = \frac{a}{1-r}$ <p><u>Example 6:</u>                      Find the sum to infinity of : <math>1 + (1/2) + (1/2)^2 + (1/2)^3 + \dots</math></p>	<p><u>Example 1:</u>                      Fourth term = <math>a+3d</math>                      15<sup>th</sup> term = <math>a + 14d</math>                      100<sup>th</sup> term = <math>a + 99d</math></p> <p><u>Example 2:</u>                      For the progression: 3, 7, 11, 15, .....                      Find the 50<sup>th</sup> term.  <math>a = 3 ; d = 7 - 3 = 4</math>  <math>a_{50} = a + 49d = 3 + 49(4) = 199</math>                      Find the sum of the first 20 terms of the above sequence :  <math>S_{20} = (20/2)[2(3) + (20-1)(4)]</math>  <math>= (10)[6+57] = 630</math>                      It is an A.P. of first term <math>a=1</math> and common difference <math>d = 2</math>  <math>S = (n/2)[2(1) + (n-1)(2)]</math>  <math>= (n/2)[2n] = n^2</math></p> <p><u>Example 4:</u>                      7<sup>th</sup> term = <math>ar^6</math>                      40<sup>th</sup> term = <math>ar^{39}</math></p> <p><u>Example 5:</u>                      For the sequence 3, 6, 12, .....                      Find the 10<sup>th</sup> term.  <math>a = 3 , r = 6/3 = 2</math>  <math>a_{10} = ar^9 = 3(2)^9</math>                      Find the sum of the first 15 terms:  <math>S_n = a \times \frac{r^n - 1}{r - 1} ; S_{15} = 3 \times \frac{2^{15} - 1}{2 - 1}</math>  <math>S_{15} = 3(2^{15} - 1)</math></p> <p><math>\Rightarrow a = 1 , r = 1/2 &lt; 1</math>  <math>S = a/(1-r) = 1/(1-1/2) = 2</math></p>