For comments, corrections, etc…Please contact Ahnaf Abbas: ahnaf@mathyards.com

Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

Hyperbolic Functions   Handout #4

1. Uses of the hyperbolic functions:
   a. Engineering applications (catenary curves)
   b. Finding certain antiderivatives.
   c. Solving differential equations, such as
      $y''(x) = y(x)$, which has solution $y(x) = A\sinh(x) + B\cosh(x)$
      vs. $y''(x) = -y(x)$, which has solution $y(x) = A\sin(x) + B\cos(x)$

2. Definition of the hyperbolic functions:

$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\tanh x = \frac{\sinh x}{\cosh x}\left(= \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}\right)$	$\coth x = \frac{\cosh x}{\sinh x}\left(= \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1}\right)$
$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$	$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$

3. Some identities involving hyperbolic functions:

$\cosh^2 u - \sinh^2 u = 1$ Note: if $x = \cosh u$ and $y = \sinh u$, then $x^2 - y^2 = 1$ is the equation of a hyperbola	$\sinh(i\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2}$ $= \frac{(\cos\theta + i\sin\theta) - (\cos(-\theta) + i\sin(-\theta))}{2}$ $= i\sin\theta$
$\sinh 2u = 2\sinh u \cosh u$	$\cosh 2u = 2\cosh^2 u - 1$
$\cosh(s + t) = \cosh s \cosh t + \sinh s \sinh t$	$\sinh(s + t) = \sinh s \cosh t + \cosh s \sinh t$

4. Osbornes's Rule: a trigonometry identity can be converted to an analogous identity for hyperbolic functions by expanding, exchanging trigonometric functions with their hyperbolic counterparts, and then flipping the sign of each term involving the product of two hyperbolic sines. For example, given the identity $\cos(x - y) = \cos x \cos y + \sin x \sin y$, Osborne's rule gives the corresponding identity $\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$.

5. Graphs of the hyperbolic functions: