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Remainder Theorem Handout #2

Topic	Interpretation

Euclidean division

When you divide $f(x)$ by g(x) , you'll obtain the quotient and the remainder of the division.

$$f(x) = divisior \cdot quotient + remainder$$

$$f(x) = d(x) \cdot Q(x) + R(x)$$

The Remainder Theorem

If a number c is substituted for x in a polynomial $f(x)$, then the result $f(c)$ is the remainder.

This means that you can find the remainder of division of any polynomial by polynomials of the form x – c without performing the actual division.

That would be obtained by dividing $f(x)$ by $x - c$.

That is, if

$f(x) = (x - c) \bullet Q(x) + R$, then

$f(c) = (c - c) \bullet Q(x) + R$
$= 0 \bullet Q(x) + R$
$= R$

Factor Theorem

Let f be a polynomial function. Then $x-c$ is a factor of $f(x)$ if and only if $f(c)=0$. In other words, $x-c$ is a factor of $f(x)$ if and only if the remainder is zero.

<u>Example1</u>: dividing 13 by 4 :

13 = 4 x 3 + 1

divisor **=** 4 , quotient = 3 , remainder=1

<u>Example2</u>:

The remainder of division of
$f(x)= x^3 - 3x^2 + x - 5$ by x+1 is
$R= f(-1) = (-1)^3 - 3(-1)^2 + (-1) - 5$
$= -1 -3 -1 - 5$
$= -10$

The remainder of division of
$f(x)= x^3 - 3x^2 + x - 5$ by x - 1 is
$R= f(1) = (1)^3 - 3(1)^2 + (1) - 5$
$= 1 -3 + 1 - 5$
$= -6$

<u>Example3:</u>

Determine whether 4 is a root of $f(x)=0$, where
$f(x) = x^3 - 6x^2 + 11x - 6$.
We use synthetic division and the remainder theorem to find $f(4)$.

4	1	–6	11	–6
		4	–8	12
	1	–2	3	6

R = 6

Using remainder theorem:
$f(x) = 4^3 - 6(4)^2 + 11(4) - 6$
$= 64 - 96 + 44 - 6$
$= 6$

Since $f(4) \neq 0 \Rightarrow 4$ is not a root of $f(x)=0$

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Summary

f(c) = 0

⇔ the remainder of division of f(x) by (x – c) is 0.

⇔ c is a root of the equation f(x) = 0

⇔ x – c is a factor of f(x)

Theorem

A polynomial function cannot have more real zeros than its degree.

Rational Root Theorem

$f(x)=a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_0$

, where all the coefficients are integers.
if *f* has a rational root, then the possible rational roots are $\pm \dfrac{factors\ of\ a_0}{factors\ of\ a_n}$.

Example4:

$f(x) = x^3 - 2x^2 + x - 2$.

$f(2) = 2^3 - 2(2)^2 + 2 - 2 = 0$, hence

- 2 is a root of the equation : $x^3 - 2x^2 + x - 2 = 0$
- x – 2 is a factor of $f(x) = x^3 - 2x^2 + x - 2$

To find the other factor , perform long division of f(x) by x – 2 to get quotient = $x^2 + 1$ and of course remainder = 0

Hence $f(x) = (x - 2)(x^2 + 1)$

Example5:

Find the possible rational roots of

$f(x) = x^3 - 3x - 2$

possible rational roots are

$\pm \dfrac{factors\ of\ a_0}{factors\ of\ a_n} .= \pm \dfrac{1, 2}{1} = \pm 1 , \pm 2$

Steps for Finding the Real roots of a Polynomial

1. Use the degree of the polynomial to determine the maximum number of zeros.
2. If the polynomial has integer coefficients, use the Rational Root Theorem to identify those rational numbers that potentially could be zeros.
   a. Use substitution, synthetic division, or long division to test each potential rational zero
   b. Each time that a zero (and thus a factor) is found, repeat Step 2 on the depressed equation.
3. In attempting to find the zeros, remember to use (if possible) the factoring techniques that you already know (special products, factoring by grouping,…..

Example6: Find all the roots (or zeros) of

$f(x) = x^3 - 3x - 2$

Possible rational roots are ± 1 , ± 2 (Example4)

Using the remainder theorem,

f(-1) = 0 ⇒ $x = -1$ is root ⇒ x + 1 is a factor

f(2) = 0 ⇒ $x = 2$ is root ⇒ x - 2 is a factor

Choosing to work with $x = -1$ we have

$$\begin{array}{r} x^2 \quad - x - 2 \\ x+1 \overline{\big) x^3 + 0x^2 - 3x - 2} \\ \underline{-\left(x^3 + x^2\right)} \\ - x^2 - 3x - 2 \\ \underline{-\left(- x^2 - x\right)} \\ - 2x - 2 \\ \underline{-\left(- 2x - 2\right)} \\ 0 \end{array}$$

So $f(x) = x^3 - 3x - 2 = (x+1)(x^2 - x - 2)$

Factoring the quadratic quotient we have,

$f(x) = x^3 - 3x - 2 = (x+1)(x+1)(x-2)$

So the roots are x = –1, –1, and 2