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Integration III   Handout #17

Topic	Interpretation
Integration by parts $$\int u\,dv = uv - \int v\,du$$ Example: $$\int x\,e^x\,dx$$	Let u = x $\Rightarrow$ du = dx and dv = e^x dx $\Rightarrow$ v = $\int e^x dx$ = e^x $$\int x\,e^x\,dx = uv - \int v\,du = xe^x - \int e^x\,dx$$ $$= xe^x - e^x + C$$
Integration by partial Fractions *Reminder:* $$\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$$ Example: $$\int \frac{dx}{x-2} = \ln(x-2)$$ $$\int \frac{p(x)}{q(x)}\,dx$$ degree p(x) < degree q(x) $$= \frac{a}{x-x_1} + \frac{b}{x-x_2} + \dots$$ Example: $$\int \frac{2x+1}{x^2+5x+6}\,dx$$	$$\frac{2x+1}{x^2+5x+6} = \frac{a}{x+2} + \frac{b}{x+3}$$ Multiplying by x^2 + 5x + 6 = (x +2)(x +3) : 2x + 1 = a(x +3) + b(x +2) Choose x = - 2 2(-2) + 1 = a(1) + b(0) $\Rightarrow$ a = -3 Choose x = - 3 2(-3) + 1 = a(0) + b(-1) $\Rightarrow$ b = 5 $$\frac{2x+1}{x^2+5x+6} = \frac{-3}{x+2} + \frac{5}{x+3}$$ $$\int \frac{2x+1}{x^2+5x+6}\,dx = \int \frac{-3}{x+2}\,dx + \int \frac{5}{x+3}\,dx$$ $$= -3\ln(x+2) + 5\ln(x+3) + C$$
If degree p(x) $\geq$ degree q(x)	Try Long division.

Example: $\int \dfrac{t^2+1}{3t+t^3} dt$ Although degree of t^2+1 is $<$ degree $3t + t^3$; No need

for partial fractions Note that if $u= 3t + t^3 \Rightarrow du=(3 + 3t^2)dt = 3(1+ t^2)dt$

$\Rightarrow \dfrac{du}{3} = (1+t^2)dt$ substituting in the integral :

$\int \dfrac{t^2+1}{3t+t^3} dt = \int \dfrac{du/3}{u} = \dfrac{1}{3}\int \dfrac{du}{u} = \dfrac{1}{3}\ln u + C = \dfrac{1}{3}\ln(3t+t^3) + C$

Example: Show that $\dfrac{x^3+2}{x^2-1} = x + \dfrac{x+2}{x^2-1}$

Using this result and the method of partial fractions ,determine :

$\int \dfrac{x^3+2}{x^2-1} dx$

$x + \dfrac{x+2}{x^2-1} = \dfrac{x^3-x+x+2}{x^2-1} = \dfrac{x^3+2}{x^2-1}$

$\int \dfrac{x^3+2}{x^2-1} dx = \int x\, dx + \int \dfrac{x+2}{x^2-1} dx$

The first one : $\int x\, dx = \dfrac{x^2}{2}$; The second one by partial fractions:

$\dfrac{x+2}{x^2-1} = \dfrac{a}{x-1} + \dfrac{b}{x+1}$; Multiplying by x2-1 = (x-1)(x+1)

x+2 = a(x+1) + b(x-1)

Choose x = 1 : 1 + 2 = a(2) +b(0) $\Rightarrow$ a = 3/2

Choose x = - 1 : -1 + 2 = a(0) + b(-2) $\Rightarrow$ b = -1/2

$\dfrac{x+2}{x^2-1} = \dfrac{\frac{3}{2}}{x-1} + \dfrac{\frac{-1}{2}}{x+1} \Rightarrow \int \dfrac{x+2}{x^2-1} dx = \dfrac{3}{2}\int \dfrac{dx}{x-1} - \dfrac{1}{2}\int \dfrac{dx}{x+1}$

$= \dfrac{3}{2}\ln(x-1) - \dfrac{1}{2}\ln(x+1)$

$\int \dfrac{x^3+2}{x^2-1} dx = \int x\, dx + \int \dfrac{x+2}{x^2-1} dx = \dfrac{x^2}{2} + \dfrac{3}{2}\ln(x-1) - \dfrac{1}{2}\ln(x+1) + C$