For comments, corrections, etc…Please contact Ahnaf Abbas: ahnaf@mathyards.com
Sharjah Institute of Technology

معهد الشارقة للتكنولوجيا
SHARJAH INSTITUTE OF TECHNOLOGY

IntegrationI Handout #15

Topic	Interpretation
Anti-Derivative F'(x) = f(x) Example: f(x)=2x is the derivative of F(x)= x^2 here,F'(x) = f(x) F(x) = x^2+1 ;F'(x) = 2x	F(x) is the antiderivative of f or the primitive of f . x^2 is the antiderivative of 2x x^2+1 is the antiderivative of 2x All antiderivatives of a certain function differ only by a constant.
Indefinite Integral $\int f(x)dx$ Example: $\int 2xdx =$ x^2 + C	f(x) is the derivative of what function? Derivative of x^2 + C is 2x
Basic rules: $\int x^k dx = \frac{x^{k+1}}{k+1}$ + C ; $k \neq 1$ Example: $\int x^5 dx$ $\int \sqrt{x}dx$ = $\int x^{\frac{1}{2}}dx$ *Things to Remember:* $\int kdx$ = kx + C $\int kf(x)dx = k\int f(x)dx$	$= \frac{x^6}{6} + C$ $= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$ + C = $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ + C = $\frac{2x^{\frac{3}{2}}}{3}$ + C Examples: $\int 5dx =$ 5x + C ; $\int dx =$ x + C $\int 4x^2 dx = 4\int x^2 dx = 4\frac{x^3}{3}$ + C
Definite Integral $\int_a^b f(x)dx = F(b) - F(a)$	$\int_0^1 x^4 dx$ = $\left.\frac{x^5}{5}\right\|_0^1$ = F(1) – F(0)

Example: $\int_0^1 x^4 dx$	$= \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$
Integration by Substitution Suppose you need to find : $\int (2x+1)^2 dx$	One way to do it is to expand $(2x+1)^2$ So it will turn into something we know based on the above rule: $(2x+1)^2 = 4x^2 + 4x + 1$ $\int (2x+1)^2 dx = \int (4x^2 + 4x + 1)dx$ $= 4\frac{x^3}{3} + 4\frac{x^2}{2} + x + C = \frac{4x^3}{3} + 2x^2 + x + C$
What about : $\int (2x+1)^{12} dx$ It is not practical to expand $(2x+1)^{12}$; So we use Substitution. Example: $\int x^3(\sqrt{x^2+1})dx$ Let $u = x^2 + 1$,then the integral becomes : $\int x^3\sqrt{u}dx$ We need to change *dx* into *du* $u = x^2 + 1 \Rightarrow du = 2x\,dx$ $\Rightarrow dx = \frac{du}{2x}$ substituting this in the above integral: $\int x^3\sqrt{u}\frac{du}{2x} = \frac{1}{2}\int x^2\sqrt{u}du$	Let $u = 2x + 1$,then the integral becomes : $\int u^{12}dx$ We need to change *dx* into *du* $u = 2x + 1 \Rightarrow du = 2\,dx$ $\Rightarrow dx = \frac{du}{2}$ substituting this in the above integral: $\int u^{12}dx = \int u^{12}\frac{du}{2} = \frac{1}{2}\int u^{12}du$ $= \frac{1}{2}\frac{u^{13}}{13} + C$; With $u = 2x + 1$: $= \frac{1}{2}\frac{(2x+1)^{13}}{13} + C = \frac{(2x+1)^{13}}{26} + C$
Now, we need to get rid of x^2 : $u = x^2 + 1 \Rightarrow x^2 = u - 1$ $\frac{1}{2}\int x^2\sqrt{u}du = \frac{1}{2}\int (u-1)\sqrt{u}du$ $= \frac{1}{2}\int u\sqrt{u}du - \frac{1}{2}\int \sqrt{u}du$ $= \frac{1}{2}\int u \times u^{\frac{1}{2}}du - \frac{1}{2}\int u^{\frac{1}{2}}du$	$\frac{1}{2}\int u^{\frac{3}{2}}du - \frac{1}{2}\frac{u^{\frac{3}{2}}}{3/2} + C = \frac{1}{2}\frac{u^{\frac{5}{2}}}{5/2} - \frac{u^{\frac{3}{2}}}{3} + C$ $= \frac{u^{\frac{5}{2}}}{5} - \frac{u^{\frac{3}{2}}}{3} + C = \frac{(x^2+1)^{\frac{5}{2}}}{5} - \frac{(x^2+1)^{\frac{3}{2}}}{3} + C$