Polynomials Division Handout #1

Topic	Interpretation
Polynomials A POLYNOMIAL IN ONE VARIABLE is a polynomial that contains only one variable. The DEGREE of a polynomial in one variable is the greatest exponent of its variable. A LEADING COEFFICIENT is the coefficient of the term with the highest degree. A **polynomial function** is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where n is a whole number {i.e. 0, 1, 2, 3, 4, …} and $a_n, a_{n-1}, \ldots, a_1, a_0$ are real number coefficients.	Example1: $\mathbf{5x^2 + 3x - 7}$ Example2: What is the degree and leading coefficient of $3x^5 - 3x + 2$? Degree = 5 , leading coefficient = 3 Example3: Find f(-2) if $f(x) = 3x^2 - 2x - 6$ $f(-2) = 3(-2)^2 - 2(-2) - 6$ $= 3(4) + 4 - 6 = 10$ EVALUATING A POLYNOMIAL FUNCTION: Simply substitute the given value of the variable into the polynomial expression Example4: Find f(m + 2) if $f(x) = 3x^2 - 2x - 6$ $f(m + 2) = 3(m + 2)^2 - 2(m + 2) - 6$ $f(m + 2) = 3(m^2 + 4m + 4) - 2(m + 2) - 6$ $f(m + 2) = 3m^2 + 12m + 12 - 2m - 4 - 6$ $f(m + 2) = 3m^2 + 10m + 2$
Let $\frac{f(x)}{g(x)}$ be a rational function where the degree of $f(x)$ is more than the degree of $g(x)$. • Then perform the long division $g(x)\overline{)f(x)}$ by determining what you must multiply the leading term of $g(x)$ by to obtain the leading term of $f(x)$. • Continue the division operation until the remainder is 0, or until the degree of the remainder is less than the degree of the divisor. If you do not obtain a remainder of 0, either you made a mistake in your division or $g(x)$ is not a factor of $f(x)$.	Example5: *Example* 4: Find the quotient and remainder for $\frac{x^3 - 7x^2 + 13x + 3}{x - 2}$. Start with $x^3 \div x = x^2$, then multiply x^2 by x-2 $x^2 - 5x + 3$ $x - 2\overline{)x^3 - 7x^2 + 13x + 3}$ $-(x^3 - 2x^2)$ $-5x^2 + 13x + 3$ $-(-5x^2 + 10x)$ $3x + 3$ $-(3x - 6)$ 9 So the quotient is $x^2 - 5x + 3$ and the remainder is $R = 9$.

Synthetic Division

Long Division

$$\begin{array}{r} x\ \ +3 \\ x+2\overline{)\,x^2+5x+6} \\ \underline{x^2+2x}\qquad \\ 3x+6 \\ \underline{3x+6} \\ 0 \end{array}$$

Synthetic Division

$$x+2 \rightarrow x-(-2)$$

$$\begin{array}{r|rrr} -2 & 1 & 5 & 6 \\ & & -2 & -6 \\ \hline & 1 & 3 & 0 \end{array}$$

quotient: 1 3

remainder: 0

$$x^2+5x+6=(x+2)(x+3)$$

Example6

Use synthetic division to find the quotient and remainder.

$$\left(-4x^5+x^4+6x^3+2x^2+50\right)\div(x-2)$$

$$\begin{array}{r|rrrrr|r} 2 & -4 & 1 & 6 & 2 & 0 & 50 \\ & & -8 & -14 & -16 & -28 & -56 \\ \hline & -4 & -7 & -8 & -14 & -28 & -6 \end{array}$$

Note: We must write a 0 for the missing term.

The quotient is $-4x^4-7x^3-8x^2-14x-28$ and the remainder is –6.