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Sharjah Institute of Technology

Handout #5

Basics V - Trigonometric Functions

Topic	Interpretation

Definitions

In fig. 3.1 :

Sine : Sinθ = $\frac{o}{h}$

Cosine: Cosθ = $\frac{a}{h}$

Tangent: Tanθ = $\frac{o}{a} = \frac{\sin\theta}{\cos\theta}$

Cotangent:

Cot$\theta = \frac{a}{o} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

Secant: sec$\theta = \frac{h}{a} = \frac{1}{\cos\theta}$

Cosecant: csc$\theta = \frac{h}{o} = \frac{1}{\sin\theta}$

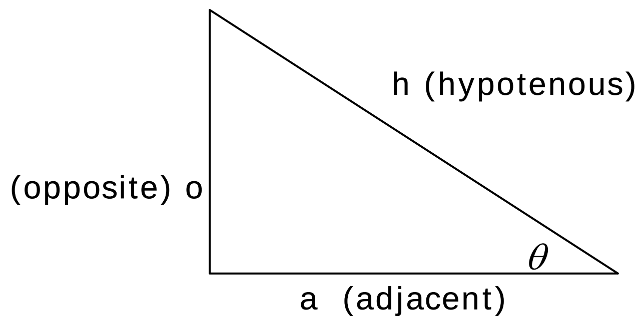

fig 3.1

Properties

$-1 \le \sin\theta \le 1$; $-1 \le \cos\theta \le 1$ so you will **never** find an angle θ such that cosθ = 2.

$-\infty \prec \tan\theta \prec +\infty$; $-\infty \prec \cot\theta \prec +\infty$

It is acceptable to have an angle θ such that tanθ = 200.

Measurements and Periodicity

Angles are measured either in degrees (°) or in radians (rd)

$\alpha^{\circ} = \alpha_{rd} \times \frac{180}{\pi}$; $\alpha_{rd} = \alpha^{o} \times \frac{\pi}{180}$

Periodicity:

Sinθ and Cosθ are periodic functions of period 2kπ

K$\in Z$ (integer)

cos(α + 2kπ) = cosα ;

sin(α + 2kπ) = sinα

tanθ and Cotθ are periodic functions of period kπ

K$\in Z$ (integer)

tan(α + kπ) = tanα ;

cot(α + kπ) = cotα

<u>Example1</u>:Convert $\frac{5\pi}{6} rd$ into degrees

$\frac{5\pi}{6} rd \times \frac{180}{\pi} = \frac{5 \times 180}{6} = 150°$

Convert 120° into radians:

$120° \times \frac{\pi}{180} = \frac{2\pi}{3} rd$

°	0	30	45	60	90	180	270	360
rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

e.g. cos(t + 6π) = cost ; sin(t + 5π) = sin (t + π + 4π) = sin(t+π) = - sint

e.g. tan(t + 6π) = tant ;

tan(t + 5π) = tant

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sin(-x) = -sin(x) cos(-x) = cos(x) tan(-x) = -tan(x) cot(-x) = -cot(x) sec(-x) = sec(x) csc(-x) = -csc(x)	sin(π/2-x) = cos(x), cos(π/2-x) = sin(x), tan(π/2-x) = cot(x), cot(π/2-x) = tan(x), sec(π/2-x) = csc(x), csc(π/2-x) = sec(x).	sin(π/2+x) = cos(x), cos(π/2+x) = -sin(x), tan(π/2+x) = -cot(x), cot(π/2+x) = -tan(x), sec(π/2+x) = -csc(x), csc(π/2+x) = sec(x).
sin(π-x) = sin(x), cos(π-x) = -cos(x), tan(π-x) = -tan(x), cot(π-x) = -cot(x), sec(π-x) = -sec(x), csc(π-x) = csc(x).	sin(π+x) = -sin(x), cos(π+x) = -cos(x), tan(π+x) = tan(x), cot(π+x) = cot(x), sec(π+x) = -sec(x), csc(π+x) = -csc(x).	**You don't have to memorize these, Use your Calculator to check them.**i.e. if you are faced with cos(t + π) choose t = 30 ;find cos 30=0.86 and cos (30+180)= cos210=-0.86;hence cos(t + 180) = -cost

Basic Relations :

$$\csc\alpha = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \tan\alpha = \frac{1}{\cot\alpha} = \frac{\sin\alpha}{\cos\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \quad 1+\tan^2\alpha = \sec^2\alpha \quad \sin\alpha = \pm\sqrt{1-\cos^2\alpha}$$

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

Double Angle Relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

Exponential Relations

Where: $i = \sqrt{-1}$; **Euler's Formula :** $e^{i\alpha} = \cos\alpha + i\sin\alpha$

$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

Derivatives:

y = sin ax	y' = a cosax	y = cos ax	y' = -a sinax
y = tanax	y' = a(1+tan^{2}ax)	y =cotax	y' = -a(1+cot^{2}x)

Integrals:

$$\int \sin \boldsymbol{axdx} = \frac{-1}{a}\text{cosax} + c \;;\; \int \cos \boldsymbol{axdx} = \frac{1}{a}\text{sinax} + c$$

$$\int \tan \boldsymbol{axdx} = \frac{-1}{a}\ln(\text{cosax}) + c \;;\; \int \cot \boldsymbol{axdx} = \frac{1}{a}\ln(\text{sinax}) + c$$