For comments, corrections, etc…Please contact Ahnaf Abbas: ahnaf@mathyards.com
Sharjah Institute of Technology

Basics II: Equations Handout #2

Topic	Interpretation
Linear equations In which all variables are raised to the power of one: Simplest form: ax + b = 0 It can be solved by moving the unknown to left side and all other terms to the right side: ax + b =0$\Rightarrow$ax = -b $\Rightarrow x = -b/a$, provided a$\neq 0$ (since division by zero is prohibited, **Reason**: try to divide 5 pencils among zero students!). General solution: **Case1**: a$\neq 0$, **b**:any real number, unique solution : $x = -b/a$ **Case2 :** a$=0$, **b**:any real number: $0 \times x = b$ Impossible, no solution. **Case3 :** a = 0 , b = 0 : $0 \times x = 0$ Infinite number of solutions	Example1: $\frac{x}{4} - 3 = \frac{x}{5} + 1$,a good practice is to get rid of the fractions : Multiplying both sides by 20 : 5x – 60 = 4x + 20 $\Rightarrow$ 5x – 4x = 20 + 60 $\Rightarrow$ x = 80 Example2: 3(2x -1) + 5x + 7 = -2x + 43 6x – 3 + 5x + 7 = -2x + 43 11x + 4 = -2x + 43 $\Rightarrow$11x + 2x =43 – 4 13x = 39 $\Rightarrow$x = 39/3 $\Rightarrow$ x = 3 Example3: 7x -11 + 3x = 10x – 2 10x = 10x -2 + 11 10x – 10x = 9 0x = 9 , impossible , you can't find a number when multiplied by 0 gives you 9 Example4: 5x = 5x , 5x – 5x = 0 , $0 \times x = 0$ Any number you put for x will satisfy the equation: $0 \times 2 = 0$, $0 \times -10 = 0$ $0 \times 93 = 0$, 2 ,-10,93,…. are all solutions The equation has infinite number of solutions
Absolute Value Equations |x| = a ; a ≥ 0 (since |x|≥ 0**)** **|x| = a $\Rightarrow$ x = $\pm$ a** Remove the absolute value and $\pm$ the answer	Example5: | x - 3 | = 5 $\Rightarrow$x – 3 = $\pm$ 5 Either x – 3 = - 5 $\Rightarrow$x = -5 + 3 = -2 OR x – 3 = 5 $\Rightarrow$ x = 5 + 3 = 8 Example6: | x | = - 7 ;No solution since |x| ≥ 0 **.**
Absolute Value inequalities |x| < a $\Rightarrow$ - a < x < a |x| > a $\Rightarrow$ x < - a ; x > a	Example7: (1.) |x| < 6 $\Rightarrow$ -6 < x < 6 (2.) | x + 2| < 7 $\Rightarrow$-7 < x+2 < +7 $\Rightarrow$-9 < x < 5 (3.) |2x – 3 | > 9$\Rightarrow$ 2x – 3 < - 9 $\Rightarrow$ 2x < - 6 $\Rightarrow$ x < -3 2x – 3 > 9 $\Rightarrow$ 2x > 12 $\Rightarrow$x > 6

Quadratic Equations

$ax^2 + bx + c = 0$

a,b and c are constants; $a \neq 0$

Can be solved by *factoring* or using the *quadratic formula* :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case1: $b^2 - 4ac > 0$; Two *distinct* real roots.

Case2 : $b^2 - 4ac = 0$; One double root $x = -b/2a$

Case3: $b^2 - 4ac < 0$; No real roots.

Example8:

(1.) $x^2 - 7x + 6 = 0$;

by factoring: $(x - 1)(x - 6) = 0$; $x = 1$; $x = 6$

By Formula : a = 1 ; b = -7 ; c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4(1)(6)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{25}}{2}$$; x = 1 ; x = 6 .

(2.) $x^2 - 3x + 7 = 0$;

a= 1 , b = -3 , c = 7;

$b^2 - 4ac = (-3)^2 - 4(1)(7) = 9 - 28 = -19 < 0$

No real roots for this equation.

Higher Equations

BiQuadratic Equations

A. $ax^4 + bx^2 + c = 0$

a,b,c are constants ; $a \neq 0$

set $y = x^2$,the equation becomes

$ay^2 + by + c = 0$ which is a quadratic equation.

Note that $y = x^2 \geq 0$

The solution of the biquadratic equation is x2 instead of x.

B. Equations of degree ≥ 3

Substitute the divisors of the constant term in the equation: f(a) = 0 , then x = a is a root of the equation and (x-a) is a factor of f(x),see Example11

$x^3 - 4x^2 - 9x + 36 = 0$

the divisors of 36:

$\pm 1, \pm 2, \pm 3, \pm 4, \ldots$.

x=- 1:

$(-1)^3 - 4(-1)^2 - 9(-1) + 36 = 40$

-1 is not a root.

Example9: $5x^4 - 3x^2 - 2 = 0$

a = 5 ,b = -3 , c = -2

$$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(5)(-2)}}{2(5)}$$

$$x^2 = \frac{3 \pm \sqrt{49}}{10}$$; $\mathbf{x^2 = -4/10}$ *No real solution*(remember $x^2 \geq 0$**)**

$\mathbf{X^2 = 1 \; ; \; x = \pm 1}$

Recall : $\boldsymbol{x^2 = a \; ; \; a > 0}$ ***then*** $\boldsymbol{x = \pm \sqrt{a}}$

Remove the square, $\pm$square root the answer.

Example10: $x^3 - 4x^2 - 9x + 36 = 0$

You may solve this by factoring by taking them two at a time:

$x^3 - 4x^2 - 9x + 36 = 0$

$x^2(x - 4) - 9(x - 4) = 0$

$(x - 4)(x^2 - 9) = 0$

Either x -4 = 0 ; x = 4

Or $x^2 = 9$; $x = \pm \sqrt{9}$; $x = \pm 3$

x= 1: $(1)^3 - 4(1)^2 - 9(1) + 36 = 24$ x= 1 is not a root. x=4 : $(4)^3 - 4(4)^2 - 9(2) + 36 = 0$ x =4 is a root. Similarly f(-3) = 0 ; -3 is a root. f(3)= 0 ; 3 is a root	Example11: $x^3 - 2x^2 - 5x + 6 = 0$ The divisors of 6 : $\pm 1, \pm 2, \pm 3, \pm 6$ $f(1) = (1)^3 - 2(1)^2 - 5(1) + 6 = 0$; 1 is a root . $f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6 = 0$; -2 is a root . $f(3) = (3)^3 - 2(3)^2 - 5(3) + 6 = 0$; 3 is a root . The roots are : 1, -2 and 3.
Square roots Equations Equations involving square roots can be solved by squaring both sides. $\sqrt{x-1} = 3$; note that **x -1**≥ 0 $\Rightarrow$ x ≥ 1 i.e valid solutions should be greater or equal to **1**. Squaring both sides : x – 1 = 9 $\Rightarrow$ x = 10 accepted being ≥ 1	Example12: $\sqrt{2x-1} = 2x-3$ Valid solutions : 2x - 1 ≥ 0 ; x $\geq \frac{1}{2}$ Squaring both sides : $2x - 1 = (2x - 3)^2$ $2x - 1 = 4x^2 - 12x + 9$ $4x^2 - 14x + 10 = 0$ Solving by factoring or by the quadratic formula : x = 1 accepted being ≥ 0.5 x = 10/4 = 2.5 accepted being ≥ 0.5
Simultaneous Equations Two equations with two unknowns. ax + by = c dx + fy = e There are many ways to solve such equations.The easiest two are by substitution and by elimination.	Example13: x +3y = 11 ------(1) 5x – 2y = 4 -------(2) From (1), x = 11 – 3y substitute this in (2) : 5(11 – 3y) – 2y = 4 $\Rightarrow$ 55 – 15y – 2y = 4 $\Rightarrow$ -17y = -51 **y = 3**. Now x = 11 -3y = 11 – 3(3) = 2 **(x,y) = (2,3)** *Checking:* substitute the values of x and

By Substitution

Find x in terms of y in any of the equations and substitute it in the other equation.
Substitution is good when you have the coefficient of at least one of the unknowns is 1.
e.g.
$x + 7y = 10$;it is easy to set
$x = 10 – 7y$;
while in $31x – 17y = 103$,
you will face
$x = (103 + 17y)/31$,
in this case use elimination for less trouble with arithmetic.
(See Example 13)

By Elimination

Is the process where you get rid of one of the unknowns.
If it is possible to make the x has same coefficient in both equations, then by subtraction, you will be able to get rid of x.

We do so by multiplying each equation with the coefficient of x in the other equation(see Example14)

By Graphing

$ax + by = c$
$dx + fy = e$
represent two equations of two straight lines, plot the two straight lines.
the point of intersection of these straight lines represents the solution of the system.

y in any of the equations ,if these values satisfy the equation ,then your solution is correct:
$x + 3y = 11$
$2 + 3(3) = 11$.

Example 14 : $5x - 3y = -6$ ------(1)
$3x + 7y = 14$ ------(2)

Multiply (1) by 3 : $15x – 9y = - 18$

Multiply (2) by 5 : $15x + 35y = 70$

Subtracting : $-9y – 35y = -18 – 70$
$-44y = -88$; $\mathbf{y = 2}$

Now substitute this in any of the equations: $5x – 3y = -6$
$5x - 3(2) = -6$
$5x - 6 = -6$
$5x = 0$; $x = 0/5 = 0$

(x,y) = (0 , 2)

Example 15: $-2x + y = -3$ ------(1)
$-x + y = 1$ ------(2)

$x = 4$, $y = 5$