Sharjah Institute of Technology

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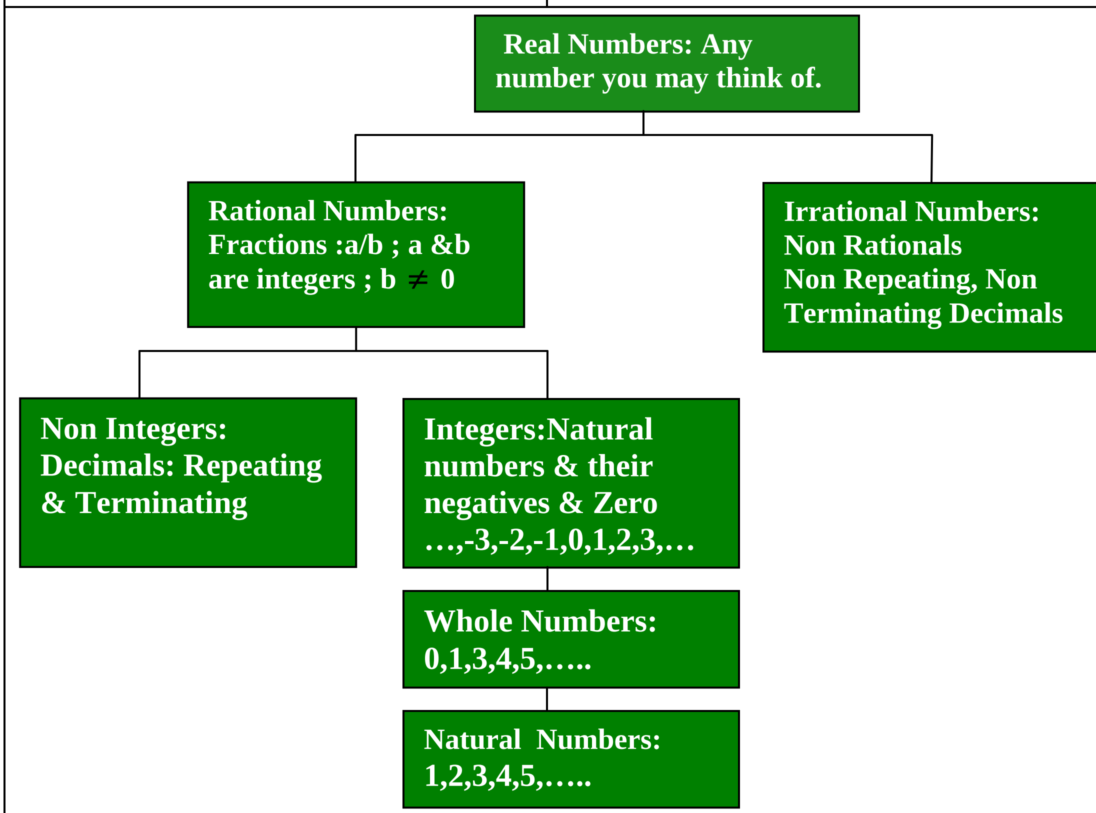
Subject : Mathematics

Basics #1

Basics I: Arithmetic

Topic	Interpretation
The Real Numbers	
N:Natural (counting) Numbers	1,2,3,4,…….
W:Whole Numbers	0,1,2,3,4,…..
Z:Integers	…..,-3,-2,-1,0,1,2,3,……
Q:Rational Numbers	All numbers of the form a/b ,where a and b are integers with $b \neq 0$ e.g. 3/2 , - 2/7 , 5 = 5/1
I:Irrational numbers	Real numbers that are not Rational e.g. π
$N \subseteq Z \subseteq Q$ $R = Q \cup I$	All the above numbers are *real* numbers.



Order of operations Follow the following order as they occur working from left to right: 1.Parenthses & square brackets 2.Powers 3.Multiplication or division 4.Addition or subtraction	$\frac{2(-2-5)^2+4(5)}{-3+4} = \frac{2(-7)^2+20}{1}$ $= \frac{2(49)^2+20}{1} = 2(49)+20 = 98+20 =$ 118 $\frac{-11-(-12)-4\times 5}{4(-2)-(-6)(-5)} = \frac{-11+12-20}{-8-(+30)} = \frac{-19}{-38}$ $= \frac{19}{38} = \frac{1}{2}$ = 0.5
Absolute Value $\lvert x\rvert = \begin{cases} -x & \text{if } x<0 \\ +x & \text{if } x\geq 0 \end{cases}$ $\lvert x \rvert \geq 0$ for every x $\lvert x - y \rvert \neq \lvert x\rvert - \lvert y\rvert$ $\lvert x + y \rvert = \lvert x\rvert + \lvert y\rvert$ for x,y having the same sign.	$\lvert -5 \rvert = 5$; $\lvert 5 \rvert = 5$ $\lvert 3 - 7 \rvert = \lvert -4 \rvert = 4$ $\lvert 3\rvert - \lvert 7\rvert = 3 - 7 = -4$ $\lvert 5+4\rvert = \lvert 9\rvert = 9$ $\lvert 5\rvert + \lvert 4\rvert = 5 + 4 = 9$
Fractions Addition $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{3}{7} - \frac{5}{8} = \frac{8\times 3 - 7\times 5}{56} = \frac{24-35}{56} = \frac{-11}{56}$
Square roots There are two numbers whose square is 25 : -5 and 5 $(-5)^2 = -5 \times -5 = 25$ $5^2 = 5 \times 5 = 25$ The *positive* one, 5, is called the square root of 25. $\sqrt{x} \geq 0$ for every real number x For $\sqrt{x}$ to exist , $x \geq 0$ $(\sqrt{x})^2 = \sqrt{x}\times\sqrt{x} = x$ $\sqrt{x+y} \neq \sqrt{x}+\sqrt{y}$ $\sqrt{x}\times\sqrt{y} = \sqrt{xy}$ $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$	$\sqrt{49} = 7$ since $7^2 = 49$ $\sqrt{-25}$ does not exist. $\sqrt{16+9} = \sqrt{25} = 5$ $\sqrt{16}+\sqrt{9} = 4+3 = 7$ $4\sqrt{5}\times 7\sqrt{3} = 28\sqrt{15}$ $\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$ $\sqrt{2}\times\sqrt{2} = 2$ $7\sqrt{2}\times 3\sqrt{2} = 21\times 2 = 42$ $\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ Estimate $\sqrt{73}$ since $8^2 = 64$ and $9^2 = 81$ $\sqrt{73}$ must be between 8 and 9.

Exponents

$x^m \times x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$\frac{1}{x^m} = x^{-m}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$2x^3 \times 5x^4 = 10x^7$

$\frac{-12x^7}{6x^5} = -2x^2$

$(2x^2)^3 = 2^3 x^6 = 8x^6$

$\frac{1}{x^3} = x^{-3}$

$\sqrt{25x^4} = 5x^{\frac{4}{2}} = 5x^2$

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

Polynomials

In the expression $2x^3$, x is called a *variable* because it can assume any number of different values.
2 is called the coefficient.
The highest power that appears in a polynomial is the degree of the polynomial.
$2x - 5x^3 + 7x^2$ is of degree 3

Like terms of a polynomial can be added or subtracted, unlike terms can not.
$9x^5 - 15x^5 = -6x^5$
$3x + 4x^2 = 3x + 4x^2$
Multiplication :

$(2x - 1)(3x + 5)$

$= (2x)(3x) + (2x)(5) -(1)(3x) -(1)(5)$
$= 6x^2 + 10x - 3x - 5 = 6x^2 + 7x - 5$

Factoring

The number 10 can be written as 5x2, 1x10,….
The numbers in each product are called *factors*, the process of writing 10 as a product of factors is called *factoring*.

Difference of two squares
$(a-b)(a+b) = a^2 - b^2$

$a^2 + b^2$ can not be factored in real numbers and is always positive; $x^2 + 5 > 0$ for every x real number.

Difference of two cubes
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of two cubes
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

<u>Example1:</u> factor $12x - 18y$
Both 12x and 18y are divisible by 6 :
$6(2x) - 6(3y) = 6(2x - 3y)$

<u>Example2</u>: $8x^3 - 9x^2 + 15x$
Each of these terms is divisible by x :
$x(8x^2) + x(-9x) + x(15) = x(8x^2 - 9x + 15)$

<u>Example3</u>: $5(4x-3)^3 - 2(4x-3)^2$
$(4x-3)^2$ is the common factor :
$(4x-3)^2 [\ 5(4x-3) - 2\ ] = (4x-3)^2(20x-17)$

<u>Example4</u>: $x^2 - 16 = x^2 - 4^2 = (x-4)(x+4)$

<u>Example5</u>: $81x^4 - 16 = (9x^2)^2 - (2^2)^2$
$= (9x^2 - 4)(9x^2+4)$
$= (3x-2)(3x+2)\ (9x^2+4)$

<u>Example6</u>: $x^3 - 8 = x^3 - 2^3$
$= (x-2)(x^2 + 2x + 4)$

Perfect Squares

$(a+b)^2 = (a+b)(a+b)$

$= a^2 + 2ab + b^2$

$(a-b)^2 = (a-b)(a-b)$

$= a^2 - 2ab + b^2$

Perfect Cubes

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Factoring Trinomial

A trinomial is a polynomial with three terms.our concern here is those of degree 2 :

$x^2 + bx + c$;

when it is possible,can be factored into two factors

$(x + m)(x+n)$

Look for the factors of the constant term *c* whose sum (or difference) is *b* ; being factors of c ,their product is c .

$x^2 + 5x + 4$

the factors of 4 whose sum is 5 are 1 and 4

$x^2 + 5x + 4 = (x+1)(x+4)$

Example7: $y^3 + 125 = y^3 + 5^3$

$= (y+5)(y^2 - 5y + 25)$

Example8: $(2x - 7)^2 = (2x)^2 - 2(2x)(7) + 7^2$

$= 4x^2 - 28x + 49$

Example9: $(q - 2)^3$

$= q^3 - 3(q^2)(2) + 3q(2^2) - 2^3$

$= q^3 - 6q^2 + 12q - 8$

Example9: Factor $x^2 - x - 6$

The factors of 6 whose sum or difference is -1 are 2 and -3 (notice their product is -6) : $(x - 3)(x+2)$

Example10: Factor $4y^2 - 11y + 6$

Here the factors should look like:

$(2y\quad)(2y\quad)$ or $(4y\quad)(y\quad)$

We do it by trial :we need 6 at the end ,i.e. (-1)(-6) ,(1)(6) ,(2)(3) or (-2)(-3)

$(2y - 1)(2y - 6) = 4y^2 - 14y + 6$ NO

$(2y - 2)(2y - 3) = 4y^2 - 14y + 6$ NO

$(4y - 3)(y - 2) = 4y^2 - 11y + 6$ YES