

Task1 : Pass Grade – P1.1

(A) The revenue of capacitors sold by your company is given by

$$R = q^3 + q^2 - 4q + 46$$
, where q is the quantity of capacitors sold daily.

Every day the company divides the revenue, of the capacitors sold, R by

$q - 3$, deposits the quotient of division in the company's bank account

and keeps the remainder of division in accountant's office.

1. Find in terms of q , the amount deposited by the company in the bank daily.
2. Use the remainder theorem to find the amount that is kept in the accountant's office every day.
3. It is found one day that the total revenue was AED 50. Use the factor theorem to solve the resulting cubic equation to find the quantity sold.
Justify your answer.

(B) The company uses rational functions as the input impedance of a RC network. There are four different synthesis methods to realize a RC network from a given rational function of input impedance. Your company uses the Foster I Synthesis Method which always require expanding $Z(s)$ in partial fractions. The technical people need your help to resolve some rational functions in order to determine the final form of each network.

You were asked to resolve the following into partial fractions:

$$1. \quad Z(s) = \frac{s^2 - 3s + 6}{s(s - 2)(s - 1)}$$

$$2. \quad Z(s) = \frac{4s - 4}{s^2 - 2s - 3}$$

$$3. \quad Z(s) = \frac{5s^2 - 2s - 19}{(s + 3)(s - 1)^2}$$

Task2 : Pass Grade – P1.2

- (A) The company needs you to check one of the new capacitors by measuring the current flowing in the capacitor at time t and the time needed to reach a certain amperage.

The current I (in amperes, **A**) flowing in a capacitor at time t (in seconds, **s**) is given by

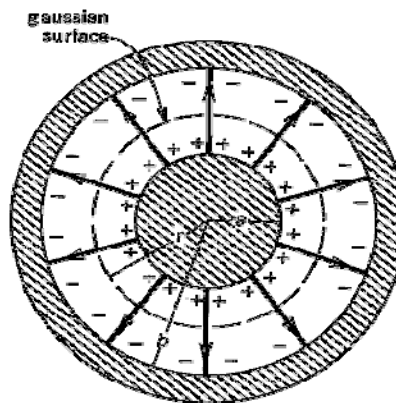
$$I = 8(1 - e^{\frac{-t}{CR}})$$

Where R is the circuit resistance (in ohms, Ω) and C is the capacitance (in Farad).

Knowing that $R = 25 \times 10^3$ ohms and $C = 16 \times 10^{-6}$ Farads, determine :

1. The current I after **0.5** seconds.
2. The time to the nearest millisecond for the current to reach **6A**.

- (B) The company is producing cylindrical capacitors which consist of an inner cylinder plate of radius a enclosed by an outer cylinder plate of radius b .



One of the machines is suspected to produce capacitors with the wrong measurement of b .

The capacitance of the cylindrical capacitor is given by

$$C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Where L is the length of the capacitor in meters and $\epsilon_0 = 8.85 \text{ pF/m}$ is the free space permittivity, a , b are in meters.

Knowing that $a = 2 \text{ cm}$, $C = 12 \text{ pF}$, and $L = 15 \text{ cm}$, calculate b .

(C) The development manager asked you to work with his team on a new project. You were faced by the following problems:

1. In an alternating current circuit, voltage $v = 5\sin \omega t$ v and current $i = 10\sin(\omega t - \pi/6)$, find an expression for the instantaneous power p At time t given that $p = vi$, expressing the answer as a difference of sines and cosines.

2. Solve the following equation for angles between 0° and 360° :

$$5\cos^2 A + 3\sin A - 3 = 0$$

3. Express $3\sin t + 5\cos t$ in the form $R\sin(t + \alpha)$ and hence solve the equation

$$3\sin t + 5\cos t = 4$$

for values of t between 0° and 360°

4. The capacitance C of a certain capacitor is given as the solution of the equation:

$$2\cosh 2C + 10\sinh 2C = 5$$

Calculate C

5. Given that $C = \sqrt{p}$, use the binomial theorem to find the percentage change in C caused by a 3% in p

Task3 : Pass Grade – P1.3

The development department is conducting an experiment on parallel capacitive circuits. They forward to you the following problem:

A parallel capacitive circuit contains **50** capacitors whose values are $C_1, C_2, C_3, \dots, C_{50}$ respectively.

The capacitance (in **Farad**) of any capacitor is equal to the capacitance of the previous one multiplied by a real constant r , i.e. for any two consecutive capacitors the relation

$$C_{n+1} = r C_n$$

is satisfied, where r is a real constant and $n = 1, 2, 3, 4, \dots, 50$:

1. Show that $C_1, C_2, C_3, \dots, C_{50}$ form a geometric series of first term C_1 and of common ratio r .
2. Knowing that $C_1 = 2 \mu F$ and $r = 3$, calculate C_{50} and determine the total capacitance of the above parallel capacitive circuit.
3. You are given now that each capacitance is given by $C_n = 2 + 3(n-1)$
 $n = 1, 2, 3, \dots, 50$.
Show that $C_1, C_2, C_3, \dots, C_{50}$ form an arithmetic series précising its first term and its common difference then determine C_{50} and calculate the sum:

$$C_1 + C_2 + C_3 + \dots + C_{50}$$

Task4 : Pass Grade – P1.4

The research department sent you the following approximation problems:

1. The capacitance C of a certain capacitor is given by the relation

$$C = \cos[\ln(1+x)] \text{ in Farad}$$

Where x is a real variable. Using the expansions of $\cos x$ and $\ln(1+x)$ expand $\cos[\ln(1+x)]$ up to the term in x^4 and then determine an approximate value of C when $x = 0.1$

2. It is found that The capacitance C of a certain capacitor is

$$C = \cosh 1.932 \text{ Farad}$$

Determine an approximate value of C using the first four terms of the power series of the function $\cosh x$